Chapter 2
Electric Potential & Capacitance

ELECTRIC POTENTIAL

Electric Potential at a point in an electric field is defined as the amount of work done in bringing a unit charge from infinity to that point without acceleration.

Suppose a charge q_o is brought at a point A in the electric field created by a charge q and W is the work done by external agent which brings the charge at A.

Then potential at point A is

$$V_A = \frac{W}{q_o}$$

SI Unit of potential is joule per coulomb which is called **volt** (V).

1 V = 1 J/C. Therefore,

Potential at a point is said to be 1 volt when a work of 1 J is done in bringing a charge of 1 C from infinity to that point without acceleration.

POTENTIAL DIFFERENCE

If a charge q is moved from a point A to a point B and work done is W, then potential difference between A and B is written as V_{AB} and it is equal to

$$V_{AB} = V_B - V_A = \frac{W}{q}$$

W is given as W = $q\left(V_B - V_A\right)$.

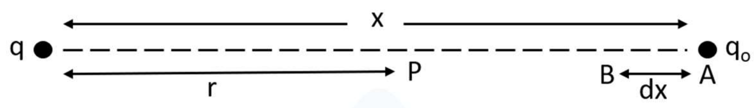
Assuming q to be positive, W will be positive if $V_B > V_A$ i.e. charge is taken from a point of lower potential to a point of higher potential. Positive work means work has to be done by external agent. Now, if $V_B < V_A$, W is negative which means work will be done by the charge i.e. charge will move by itself. Thus, we conclude

- Positive charge moves from a point at higher potential to a point at lower potential.
- Negative charge moves from a point at lower potential to a point at higher potential.

Second point can be understood by putting – q instead of q in above equation and analysing the sign of W.

POTENTIAL DUE TO A POINT CHARGE

Consider a point charge q_o placed at a point A at a distance x from a charge q as shown. Now, suppose we move this charge from A to B through small distance dx. Small amount of work done in doing so



$$dW = Fdx\cos 180^o = -Fdx$$

Angle is 180° because F and dx are opposite to each other.

Since $F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2}$, therefore $dW = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2}dx$

Now, total work done in moving this charge from $x = \infty$ to $x = r$ is

$$W = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2}dx$$

$$\Rightarrow W = -\frac{qq_o}{4\pi\varepsilon_o}\int_{\infty}^{r}\frac{1}{x^2}dx$$

$$\Rightarrow W = -\frac{qq_o}{4\pi\varepsilon_o}\int_{\infty}^{r}x^{-2}dx$$

$$\Rightarrow W = -\frac{qq_o}{4\pi\varepsilon_o}\left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r}$$

$$\Rightarrow W = -\frac{qq_o}{4\pi\varepsilon_o}\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow W = \frac{qq_o}{4\pi\varepsilon_o}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$\Rightarrow W = \frac{qq_o}{4\pi\varepsilon_o}\frac{1}{r}$$

By definition, potential at point P is $\frac{W}{q_o}$. Therefore $V = \frac{\frac{qq_o}{4\pi\varepsilon_o}\frac{1}{r}}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

Therefore, potential a distance r due to a charge q is

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}}$$

POTENTIAL DUE TO A SYSTEM OF CHARGES

As potential is a scalar quantity, so potential at a point due to a system of charges is the sum of potentials at that point due to individual charges.

ELECTROSTATIC POTENTIAL ENERGY

Work done to arrange a system of charges from infinite separation gets stored in the system in the form of electrostatic potential energy.

Consider a charge q_1 be kept at A. Let another charge q_2 be brought from infinity to point B at a distance r from it. Then, work done to bring it at P is

$$W = q_2 V$$

$$W = q_2\left(\frac{1}{4\pi\varepsilon_o}\frac{q_2}{r}\right)$$

$$W = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r}$$

This work is stored in the system in the form of electrostatic potential energy.

Thus, electrostatic potential energy of a system of two charges separated by a distance r

$$\boxed{U = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r}}$$

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Consider a charge q moving from A to B in the direction of electric field as shown. Small amount of work done is

$$dW = q(V_B - V_A)$$

$$dW = q(V - dV - V) = -qdV \quad ....(ii)$$

Also

$$dW = Fdr = qEdr \quad .....(i)$$

from (i) and (ii), we get

$$-qdV = qEdr$$

$$\boxed{E = -\frac{dV}{dr}} \quad .....(iii)$$

Therefore, E is also called potential gradient as it is equal to potential difference per unit distance.

$$dV = -\vec{E}.\vec{dr}$$

or $\boxed{\Delta V = -\int \vec{E}.\vec{dr} = \int Edr\cos\theta}$

Equation (iii) gives another unit for E which is V/m.

EQUIPOTENTIAL SURFACE

A surface on which potential is same at every point is called equipotential surface.

PROPERTIES OF EQUIPOTENTIAL SURFACE

(1) No work is done to move any charge between any two points on an equipotential surface.

Proof:

Let there be two points A and B on an equipotential surface. Then, work done to move a charge q between A and B is

$$W = q(V_B - A_A)$$
$$\because V_B = V_A$$
$$\therefore W = 0$$

(2) Electric field lines are always perpendicular to an equipotential surface.

Proof:

Let electric field makes and angle θ with the surface as shown. Now resolve E into two rectangular components:

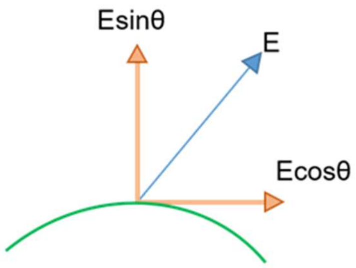
Ecosθ along the surface.

Esinθ perpendicular to the surface.

Since it is an equipotential surface so there is no flow of charge along the surface, therefore,

$E\cos\theta = 0$

$\because E \neq 0 \therefore \cos\theta = 0$ or $\theta = 90^\circ$



(3) No two equipotential surfaces can ever intersect.

(4) Equipotential surfaces are closed in the region of strong electric field and farther in the region of weak electric field.

Proof:

$\because E = -\frac{\Delta V}{\Delta r}$

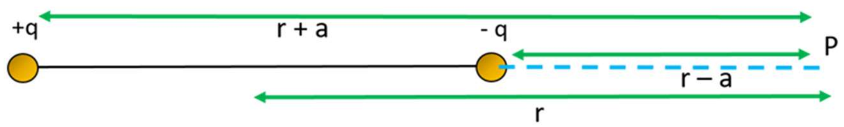
So, if ΔV is fixed $E \propto \frac{1}{\Delta r}$

POTENTIAL DUE TO DIPOLE

AT A POINT ON AXIAL LINE

Potential at P due to +q

$$V_{+q} = \frac{kq}{(r+a)}$$



Potential at P due to –q

$$V_{-q} = \frac{-kq}{(r-a)}$$

Therefore, total potential at P is

$$V_{axial} = V_{+q} + V_{-q}$$

$$\Rightarrow V_{axial} = \frac{kq}{(r+a)} + \frac{-kq}{(r-a)}$$

$$\Rightarrow V_{axial} = \frac{kq(r-a) - kq(r+a)}{r^2 - a^2}$$

$$\Rightarrow V_{axial} = \frac{kqr - kqa - kqr - kqa}{r^2 - a^2}$$

$$\Rightarrow V_{axial} = \frac{(-2aq)k}{r^2 - a^2}$$

$$\Rightarrow \boxed{V_{axial} = \frac{-kp}{r^2 - a^2}}$$

For short dipole a<<r

$$V_{axial} = -\frac{kp}{r^2}$$

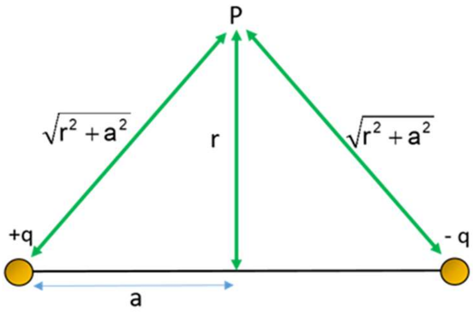
AT A POINT ON EQUATORIAL LINE

As shown in the diagram, potential at P due to +q

$$V_{+q} = \frac{kq}{\sqrt{a^2 + r^2}}$$

Potential at P due to –q

$$V_{+q} = \frac{k(-q)}{\sqrt{a^2 + r^2}}$$



Therefore, total potential at P is

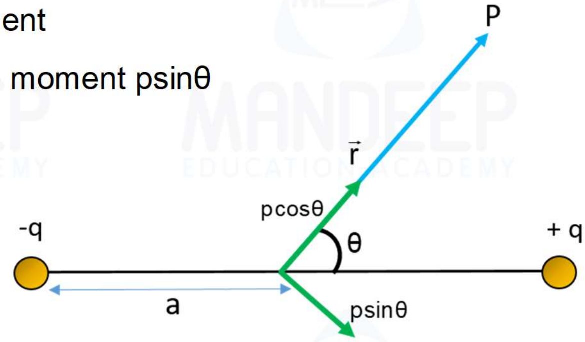
$$V_{eq} = V_{+q} + V_{-q} = 0$$

POTENTIAL AT ANY ARBITRARY POINT

Consider a point P at a distance along a line making an angle θ with the dipole axis. If we resolve $\vec{p}$ into two rectangular components as shown. Point P lies on the axial line of the dipole with dipole moment $p\cos\theta$ and on equatorial line of the dipole with the dipole moment $p\sin\theta$

$$V = \frac{kp\cos\theta}{r^2} + 0$$

$$\boxed{V = \frac{kp\cos\theta}{r^2}}$$



CAPACITANCE

The ability of a body to store charge is called capacitance.

If a charge q is stored in the body and potential of body increases by V, then Capacitance of body is given by

$$\boxed{C = \frac{Q}{V}}$$

SI unit of capacitance is coulomb per volt which is called farad (F).

$$\boxed{1\ \text{farad} = \frac{1\ \text{coulomb}}{1\ \text{volt}}}$$

Therefore, capacitance of a body is said to be one farad if its potential increases by 1 volt when a charge of 1 coulomb is given to it.

CAPACITANCE OF A SPHERICAL BODY

Consider a spherical body of radius R. If a charge q is given to it, its potential will be

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

Then the capacitance of the body is

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_o}\frac{q}{R}}$$

$$\Rightarrow \boxed{C = 4\pi\varepsilon_o R}$$

$$\therefore C = 4 \times 3.14 \times 8.85 \times 10^{-12} \times R$$

$$\Rightarrow C \approx 10^{-10} R$$

S for C to be 1 F, R = 10^{10} m which is impossible. So, 1 farad is a very big unit.

Commonly used units of capacitance

$$1\ \mu F\ (\text{microfarad}) = 10^{-6} F$$

$$1\ nF\ (\text{nanofarad}) = 10^{-9} F$$

$$1\ pF\ (\text{picofarad}) = 10^{-12} F$$

CAPACITORS

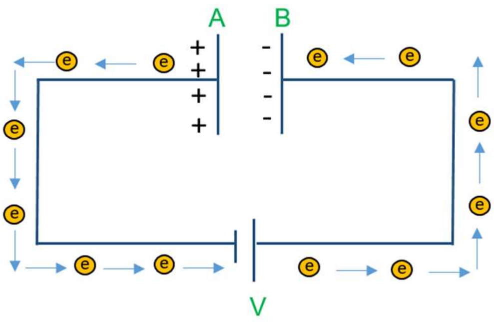
A body which is specially designed to store charge is called capacitor.

Most efficient design of a capacitor is parallel plate capacitor in which two metals plates are connected parallel to each other with some gap between them which is usually filled by some dielectric.

When capacitor is connected to battery, following things happen:

1. Electrons from plate A move from plate to battery due to the attraction of negative terminal.

2. Due to this action, plate gets positively charged.
3. Electrons from negative terminal of battery move to plate B.
4. Due to this action, plate B acquires negative charge.

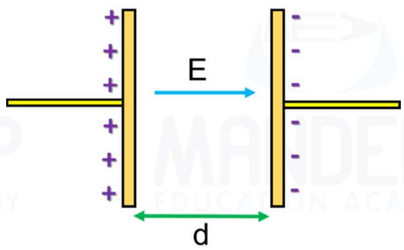


Important points to note

1. Charge acquired by one plate is always equal and opposite to the other plate i.e. if one plate acquires charge +Q, other plate acquires charge – Q.
2. The potential difference across the plates of capacitor becomes equal to potential difference of battery in fraction of seconds after connecting.
3. Charge on capacitor is then, taken as Q, not zero.
4. Capacitance of the capacitor is $C = \frac{Q}{V}$.

CAPACITANCE OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor as shown.



Let

V = potential difference between the plates

Q = charge on the capacitor

E = Electric field between plates

σ = Surface charge density of the plates

d = distance between the plates

As $C = \frac{Q}{V} = \frac{\sigma A}{V}$ $\quad [\because \sigma = \frac{Q}{A}]$

$\because V = Ed$

$\therefore C \frac{\sigma A}{Ed}$

field between plates capacitor is $E = \frac{\sigma}{\varepsilon_o}$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_o} d} \Rightarrow \boxed{C = \frac{\varepsilon_o A}{d}}$$

If there is a medium of dielectric constant k between the plates, then

$$k = \frac{\varepsilon}{\varepsilon_o} \Rightarrow \varepsilon = k\varepsilon_o \therefore \boxed{C = \frac{k\varepsilon_o A}{d}}$$

ENERGY STORED IN CAPACITOR (NOT IN SYLLABUS FOR SESSION 2023-24)

Let dW be the small amount of work by the battery to store small charge dq

So, dW = Vdq, where V is the voltage of the battery

$$\because V = \frac{q}{C}$$

$$dW = \frac{q}{C}dq$$

Then, the total work done to store charge Q is

$$\int dW = \int_0^Q \frac{q}{C}dq$$

$$\Rightarrow W = \frac{1}{C}\int_0^Q qdq$$

$$\Rightarrow W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\Rightarrow W = \frac{1}{2C}\left[Q^2 - (0)^2\right]$$

$$\Rightarrow W = \frac{Q^2}{2C}$$

This work is stored in the capacitor in the form of electrostatic energy

$$\because \boxed{U = \frac{Q^2}{2C}}$$

$$\because Q = CV$$

$$\therefore U = \frac{C^2V^2}{2C}$$

$$\therefore \boxed{U = \frac{1}{2}CV^2}$$

$$\text{or } C = \frac{Q}{V} \therefore U = \frac{Q^2}{2\frac{Q}{V}}$$

$$\boxed{U = \frac{1}{2}QV}$$

ENERGY DENSITY (U)

Energy density is energy stored in capacitor per unit volume.

$$\boxed{u = \frac{\text{Energy stored}}{\text{volume}}}$$

$$= \frac{\frac{1}{2}CV^2}{Ad}$$

$$= \frac{1}{2}\frac{\varepsilon_o A}{d}\frac{E^2 d^2}{(Ad)}$$

$$\therefore \boxed{u = \frac{1}{2}\varepsilon_o E^2}$$

DIELECTRIC POLARIZATION

When a non-polar dielectric is placed in an external electric field, it gets polarized. This phenomenon is called dielectric polarization.

- A non-polar dielectric is one in which there are no negative or positive poles of charges.
- When such material is placed in external electric field, the positive and negative centres of molecules get separated.
- This happens because electrons of bond of molecules which are towards the positive side of field gets attracted towards the positive side.
- Due to this, each molecule gets polarized and overall material gets polarized.
- Due to this an electric field gets induced in a direction opposite to that of applied field. Let this field be E_p.
- Net electric field inside the dielectric becomes $E = E_o - E_p$.
- If dielectric constant of the material is k, then $E = \frac{E_o}{k}$.

CAPACITOR WITH SLABS

CAPACITANCE OF PARALLEL PLATE CAPACITOR WITH DIELECTRIC SLAB BETWEEN THE PLATES

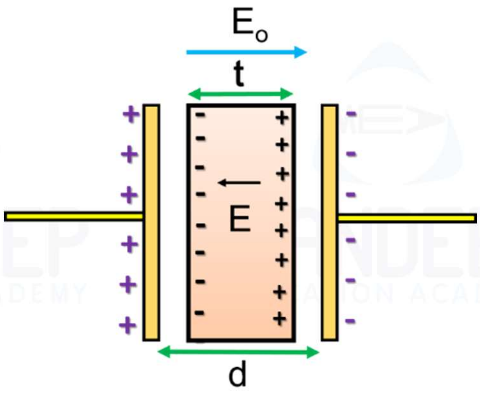
Consider a slab of thickness t inserted between the plates as shown

Potential difference between the plates is given by

$$V = E_o(d-t) + Et$$

$$\Rightarrow V = E_o(d-t) + \frac{E_o}{k}t$$

$$\Rightarrow V = E_o\left[d - t + \frac{t}{k}\right]$$



Let new capacitance be C'

$$C' = \frac{Q}{V}$$

$$\Rightarrow C' = \frac{Q}{E_o\left[d - t + \frac{t}{k}\right]}$$

$$\Rightarrow \boxed{C' = \frac{\varepsilon_o A}{d - t\left(1 - \frac{1}{k}\right)}}$$

CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH CONDUCTING SLAB BETWEEN THE PLATES

Consider a conducting slab placed between the plates of a parallel plate capacitor as shown

Since, electric field inside the conducting slab is zero, potential difference between the plates is given by

$$V = E_o(d-t) + Et$$

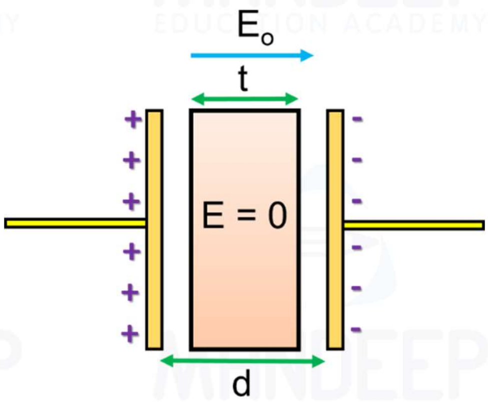
$$\Rightarrow V = E_o(d-t) + (0)t$$

$$\Rightarrow V = E_o(d-t)$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_o}(d-t)$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_o}(d-t)$$

$$\therefore C' = \frac{Q}{V} = \frac{\not{Q}}{\frac{\not{Q}}{A\varepsilon_o}(d-t)} \Rightarrow \boxed{C' = \frac{\varepsilon_o A}{d-t}}$$

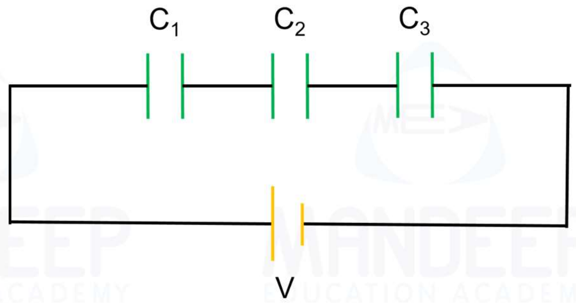


COMBINATION OF CAPACITORS

SERIES COMBINATION

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series as shown. Let potential difference across them be V_1, V_2 and V_3 and charge stored by each is Q.

If V is applied voltage, then



$$V = V_1 + V_2 + V_3$$

$$\because V = \frac{Q}{C}$$

$$\therefore V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

If equivalent capacitance is C_{eq}

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

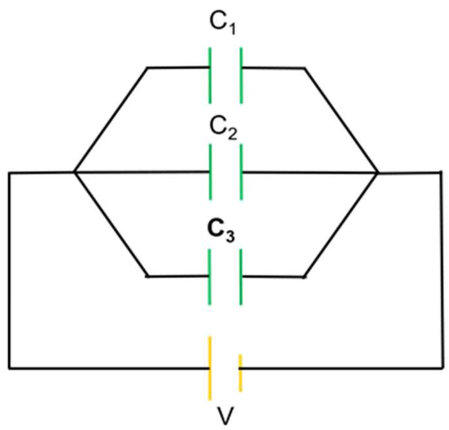
$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

PARALLEL COMBINATION

Figure shown three capacitors connected in parallel, let charge stored by each is Q_1, Q_2 and Q_3 and potential difference across each is V. If charge supplied by battery be Q, then

$$Q = Q_1 + Q_2 + Q_3$$

$$\because Q = C_{eq}V,\ C_{eq} = \text{equivalent capacitance}$$



$$Q = C_1V + C_2V + C_3V$$

$$\Rightarrow C_{eq}\cancel{V} = \cancel{V}(C_1 + C_2 + C_3)$$

$$\Rightarrow \boxed{C_{eq} = C_1 + C_2 + C_3}$$

COMMON POTENTIAL

If two capacitors of capacitances C_1 and C_2 are charged to potential V_1 and V_2 and are connected together, then, the charge flows from the capacitor at higher potential to the other at lower potential till the potential of both become equal, this equal potential is called common potential.

Since total charge before and after remains same, therefore

$$C_1V + C_2V = C_1V + C_2V$$

$$\Rightarrow \boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

LOSS OF ENERGY ON SHARING OF CHARGES

When charge is shared between the capacitors, energy is lost in the form of heat

Total energy before sharing

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

total energy after sharing

$$U_f = \frac{1}{2}(C_1 + C_2)V^2$$

$\therefore$ Heat loss, $\Delta U = U_i - U_f$

$$\Delta U = \frac{1}{2}\left\{C_1V_1^2 + C_2V_2^2 - (C_1 + C_2)V^2\right\}$$

$$\Delta U = \frac{1}{2}\left\{C_1V_1^2 + C_2V_2^2 - (C_1 + C_2)\left[\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2}\right]\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}\left\{\frac{C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2}\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}\left\{\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}C_1C_2\left\{\frac{V_1^2 + V_2^2 - 2V_1V_2}{C_1 + C_2}\right\}$$

$$\Rightarrow \boxed{\Delta U = \frac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}}$$