Chapter 3 Current Electricity

Current Electricity

It is the branch of physics in which we study the behaviour of moving charges.

Electric current (I)

Rate of flow of charge is called electric current.

If a charge q flows through any cross section of a conductor in time t, then electric current,

$$\boxed{I = \frac{q}{t}}$$

SI unit of electric current is Cs^{-1} (coulomb per second), this unit is called **ampere (A).**

Definition of 1 ampere: Current through a conductor is said to be one ampere when a charge of 1 coulomb flows through any cross section of the conductor in 1 second.

Types of current: There are two types of electric currents, **alternating current**, which changes its magnitude and direction in a sinusoidal manner with time and **direct current** which remains constant in both magnitude and direction. We will focus on direct current in this chapter. We have a separate chapter (chapter 7) for studying alternating current.

Drift velocity

In a wire, which is not connected to battery, electrons move in random manner due to their thermal energies. When the battery is connected all electrons start drifting towards the positive end of the battery.

We may define drift velocity as the average velocity with which electrons get drifted towards the positive terminal of the battery under the influence of an external electric field.

Let the initial velocities of electrons (in the absence of battery) be $u_1, u_2, u_3 .... u_n$, then, $\frac{u_1 + u_2 + u_3 .... + u_n}{n} = 0$. When the battery is applied, acceleration of each electrons is $a = \frac{eE}{m}$.

When electrons move in a conductor, they keep colliding with the heavy ions present in it and come to a momentary rest. Time gap between two successive collisions is called relaxation time (τ).

Thus, if $v_1, v_2 .... v_n$ be the final velocities of electrons then, by definition, drift velocity is

$$v_d = \frac{v_1 + v_2 ....+ v_n}{n}.$$

Since, $v_1 = u_1 + a\tau_1, v_2 = u_2 + a\tau_2, v_3 = u_3 + a\tau_3 ....v_n = u_n + a\tau_n$.

Therefore v_d becomes

$$v_d = \frac{(u_1 + a\tau_1) + (u_2 + a\tau_2) + (u_3 + a\tau_3).........+(u_n + a\tau_n)}{n}$$

$$\Rightarrow v_d = \left(\frac{u_1 + u_2 ....+ u_n}{n}\right) + a\left(\frac{\tau_1 + \tau_2 ....+ \tau_n}{n}\right)$$

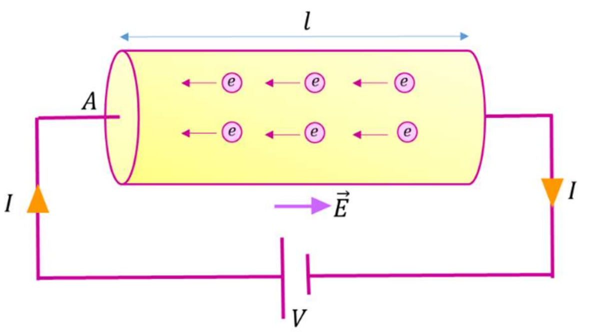
Or $\boxed{v_d = \frac{eE}{m}\tau}$,

where τ is average relaxation time.

Relation between current and drift velocity

Consider a conductor of length ℓ and area of cross section A connected to battery of potential difference V.

Then, volume of the conductor is $A\ell$.



If number density of electrons in the conductor (number of electrons per unit volume) is n, then total number of electrons in conductor is $A\ell n$.

Hence, total charge is, $q = A\ell ne$.

Therefore, current in the conductor is given by $I = \frac{q}{t} \Rightarrow I = \frac{A\ell ne}{\left(\frac{\ell}{v_d}\right)}$. Or $\boxed{I = Anev_d}$.

Ohm's law

Statement. ***It states that current flowing through a conductor is directly proportional to the potential difference applied across its ends provided the physical conditions are constant.***

As potential is work done by battery to move one coulomb of charge once around a complete circuit. So, if potential difference is more this means that battery will provide more energy to one coulomb charge and hence, the rate of low of charge i.e. current, increases. If V potential difference is applied across the end of a conductor and a current I flows through it, then $V \propto I$

Or $\qquad V = IR \qquad .... (i)$

Resistance: In the equation V = IR, r is called resistance of the material of the conductor.

It is a physical quantity that is concerned with the opposition to the flow of current through a conductor. More the resistance, more the opposition to the flow of current.

$\because V = IR$

$\therefore R = \frac{V}{I}$

Therefore, **SI unit of resistance** is volt per ampere (VA^{-1}), this unit called **ohm (Ω).**

Resistance of a conductor is said to be **1 ohm** when a current of one ampere flows through the conductor when a potential difference of 1 ohm is applied across its ends**.**

Dimensions of resistance are $\left[MA^{-2}L^{2}T^{-3}\right]$.

<u>**Proof of Ohm's law:**</u>

$\because I = Anev_d$ and $v_d = \frac{eE}{m}\tau$

$\therefore I = Ane\left(\frac{eE}{m}\tau\right)$

$\Rightarrow I = \frac{Ane^2E}{m}\tau$

$\Rightarrow I = \frac{Ane^2}{m}\left(\frac{V}{\ell}\right)\tau$

$\Rightarrow V = \frac{m\ell}{Ane^2\tau}I$ ....(ii)

If physical conditions are constant $\frac{m\ell}{Ane^2\tau}$ is constant. Therefore, $V \propto I$.

Comparing (i) and (ii), we get $\boxed{R = \frac{m\ell}{Ane^2\tau}}$ .... (iii)

Factors affecting resistance of a conductor

Resistance of a conductor depends upon following factors

i. $R \propto \ell$

ii. $R \propto \frac{1}{A}$

Combining (i) and (ii), we get $R \propto \frac{\ell}{A}$ Or $\boxed{R = \rho\frac{\ell}{A}}$ .... (iv)

Where ρ is called the **resistivity** of the conductor. It is the **specific resistance** of the material of the conductor and depends only on the nature of the material of the conductor.

Combining (iii) and (iv) we get $\rho = \frac{m}{ne^2\tau}$.

SI **unit of resistivity** is Ωm.

If $\ell = 1$ unit and A = 1 unit, then $R = \rho$.

Therefore, resistivity is equal to resistance of a conductor when length of conductor is 1 unit and area of cross section is 1 square unit.

Conductance

- Reciprocal of resistance is called conductance.
- $\boxed{G = \frac{1}{R}}$
- SI unit of conductance is $(\text{ohm})^{-1}$ or mho or siemens(S).
- Symbol of conductance is G.

Conductivity

- Reciprocal of resistivity is called conductivity.
- $\boxed{\sigma = \frac{1}{\rho}}$
- SI unit of conductivity is $\Omega^{-1}m^{-1}$ or $\text{mho}\,m^{-1}$ or Sm^{-1}.
- Symbol of conductivity is σ.

Current density (J)

- Current flowing per unit area of cross section of a conductor is called current density
- $\left(J = \frac{I}{A}\right)$.
- SI unit of current density it Am^{-2}.
- Symbol of current density is J.

Microscopic or vector form of ohm's law

$$\because J = \frac{I}{A} \therefore J = \frac{Anev_d}{A} \Rightarrow J = ne\left(\frac{eE}{m}\tau\right)$$

$$\Rightarrow J = \frac{ne^2\tau}{m}E$$

or $\boxed{\vec{J} = \sigma\vec{E}}$ .... (v)

Equation (v) is called microscopic form of ohms law or vector form of Ohm's law.

Electron mobility (μ_e)

Drift velocity per unit electric field is called electron mobility.

i.e. $\boxed{\mu_e = \frac{v_d}{E}}$. Its value represents how mobile a charge carrier is (i.e. how easily it can travel).

If a charged particle acquires higher drift velocity on application of small electric field, the its mobility is high.

SI unit of mobility is $ms^{-1}N^{-1}C$.

Temperature dependence of resistivity

Metals

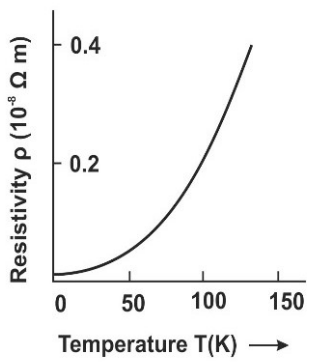
Since, resistivity, $\rho = \frac{m}{ne^2\tau}$ i.e. it is inversely proportional to relaxation time. When we increase the temperature, kinetic energy of electrons increases and they collide more frequently with ions so their relaxation time decreases and hence resistivity of metal increases. If

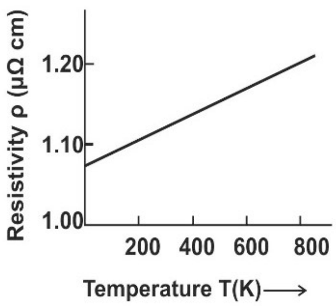
ρ_o = resistivity at $0°C$, ρ_t = resistivity at $t°C$, then

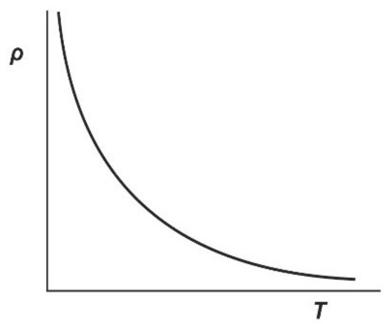
$\boxed{\rho_t = \rho_o(1+\alpha t)}$

where α is called temperature coefficient of resistivity. It is constant for a material for a given range of temperature. SI unit of α is K^{-1}. It is numerically equal to change in resistivity per unit original resistivity per degree rise in temperature.

Alloys and semiconductors





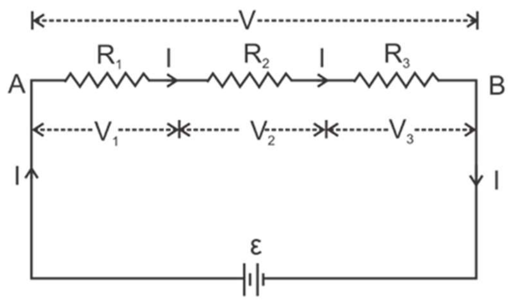


Alloys like Nichrome (which is an alloy of nickel, iron and chromium) exhibit a very weak dependence of resistivity with temperature. Manganin and constantan have similar properties. These materials are thus widely used in wire bound standard resistors since their resistance values would change very little with temperatures. Unlike metals, the **resistivities of semiconductors** decrease with increasing temperatures. Graphical variation of resistivity for metals, alloys and semiconductors is shown below.

Combination of resistors (not in syllabus since)

Series combination

Two resistors are said to be in series if only one of their end points is joined. If a third resistor is joined with the series combination of the two, then all three are said to be in series. Clearly, we can extend this definition to series combination of any number of resistors. Consider two resistors R_1 and R_2 in series. The charge which leaves R_1 must be entering R_2.



Since current measures the rate of flow of charge, this means that the same current I flows through R_1 and R_2

By Ohm's law: Potential difference across $R_1 = V_1 = IR_1$, and

Potential difference across $R_2 = V_2 = IR_2$.

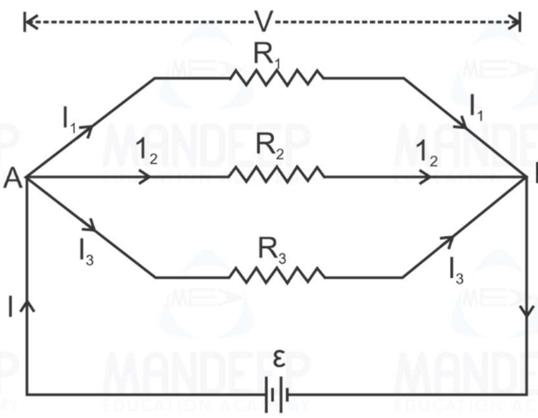
The potential difference V across the combination is $V_1 + V_2$.

Hence, $V = V_1 + V_2 = I(R_1 + R_2)$

This is as if the combination had an equivalent resistance R_{eq}, which by Ohm's law is

$$\boxed{R_{eq} = R_1 + R_2}$$

This obviously can be extended to a series combination of any number n of resistors $R_1, R_2, \ldots\ldots\ldots, R_n$. The equivalent resistance R_{eq} is $\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots\ldots\ldots}$



Parallel combination

Consider now the parallel combination of two resistors (Fig. 3.15). The charge that flows in at A from the left flows out partly through R_1 and partly through R_2. The currents I, I_1 and I_2 shown in the figure are the rates of flow of charge at the points indicated. Hence,

$$I = I_1 + I_2$$

The potential difference between A and B is given by the Ohm's law applied to R_1

$$V = I_1 R_1$$

Also, Ohm's law applied to R_2 gives

$$V = I_2 R_2$$

$$\therefore I = I_1 + I_2$$

$$\Rightarrow \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

Or $\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$

If n resistors are connected in parallel, then,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots\ldots\ldots\ldots\ldots + \frac{1}{R_n}}$$

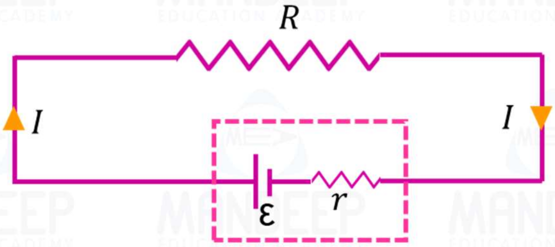
Internal resistance, terminal potential difference and emf of a cell

Internal resistance. It is the resistance offered by material of the cell. When the cell is not used in a circuit and no current is drawn from it, potential difference between its ends is called its emf.

When some current is drawn from the cell, some part of the emf is used to overcome its own internal resistance, so the potential difference across the external component is less than emf of the cell. This potential difference is called terminal potential difference.

Let ε be emf of the cell, V be the terminal potential difference, r be the internal resistance, R be external resistance and I be the current flowing in the circuit then, potential drop across internal resistance is Ir. Therefore, potential drop across external resistance is,

$\boxed{V = \varepsilon - IR}$ .... (vi)



$$\Rightarrow Ir = \varepsilon - V$$

$$\Rightarrow r = \frac{\varepsilon - V}{I}$$

$$\Rightarrow r = \frac{\varepsilon - V}{\frac{V}{R}}$$

$$\Rightarrow r = \left(\frac{\varepsilon - V}{V}\right) \times R$$

Or $\boxed{r = \left(\frac{\varepsilon}{V} - 1\right) \times R}$

<u>Charging</u> During charging of a cell, current flows in reverse direction with the help of external agency, so the terminal potential difference becomes

$\boxed{V = \varepsilon + IR}$

Combination of cells

Like resistors, cells can also be connected in series and parallel combination.

Series combination

Consider two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series. If V_1 and V_2 be the terminal potential differences of the two cells, then $V = V_1 + V_2$

$$\Rightarrow V = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$

$$\Rightarrow V = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

Comparing this with $V = \varepsilon_{eq} - Ir_{eq}$ we get

$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$

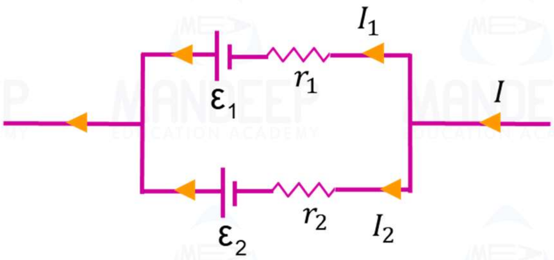
This result can be extended to series combination of n cells as $\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \ldots\ldots\ldots\ldots \varepsilon_n}$

Parallel combination

If two cells are connected in parallel, terminal potential difference across them is same but current is different,

∴ total current

$$I = I_1 + I_2$$



$$\Rightarrow I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_1 - V}{r_2}$$

$$\Rightarrow I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\Rightarrow V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$

$$\Rightarrow V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

Comparing this with $V = \varepsilon_{eq} - I r_{eq}$ we get

$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}$$

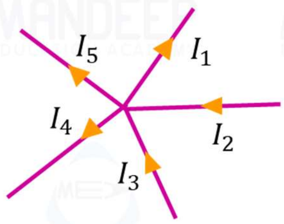
This result can be extended to parallel combination of n cells as

$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} \ldots\ldots\ldots\ldots + \frac{\varepsilon_n}{r_n}}$$

Kirchhoff's laws

1st law (junction rule)

The algebraic sum of currents meeting at a junction is 0.



In the figure shown above $I_2 + I_3 = I_1 + I_4 + I_5$ or $(I_2) + (I_3) + (-I_1) + (-I_4) + (-I_5) = 0$

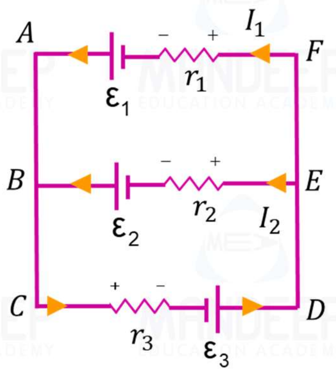
This law is the result of conservation of charge. As no charge can accumulate at a junction, so the amount of charge entering a junction per unit time is equal to amount of charge leaving junction per unit time.

2nd law (loop rule)

The algebraic sum of potential drops across all the components in a closed loop of an electric circuit is zero. This result is direct result of law of conservation of energy.

<u>Steps to solve circuits</u>

1) Assume unknown currents in the given circuit and show their directions by arrows.
2) Choose any loop and find the algebraic sum of voltage drops plus the algebraic sum of emfs in that loop and put it equal to zero.
3) Write equations for as many loops as the number of unknown quantities. Solve the equations to find the unknown quantities.
4) If the value of assumed current comes out to be negative, it means that the actual direction of current is opposite to that of assumed direction.



<u>Example:</u> In loop AFEBA : $-\varepsilon_1+I_1r_1-I_2r_2+\varepsilon_2=0$

In loop BEDCB: $-\varepsilon_2+I_2r_2-\varepsilon_3+I_3r_3=0$

Wheatstone bridge

Wheatstone bridge is a circuit which is used to measure accurately an unknown resistance.

Principle. It states that when the bridge is balanced (i.e. when $I_g=0$), the product of resistances of opposite arms is equal.

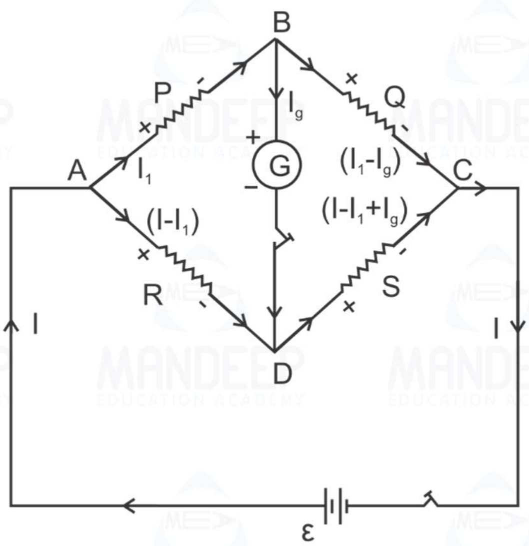
Applying Kirchhoff's second law to loop ABDA, we get

$I_1P+I_gG-(I-I_1)R=0$

SInce $I_g=0$

$\therefore I_1P-(I-I_1)P=0$

$\Rightarrow I_1P=(I-I_1)R$ ....(i)



Applying second law in loop BCDB, we get

$(I_1-I_g)Q-(I-I_1+I_g)S-I_gG=0$

$\because I_g=0$

$\therefore I_1Q-(I-I_1)S=0$

$\Rightarrow I_1Q=(I-I_1)S$ ....(ii)

From (i) and (ii) we get

$$\boxed{\frac{P}{Q}=\frac{R}{S}}$$

Or $\boxed{PS = QR}$