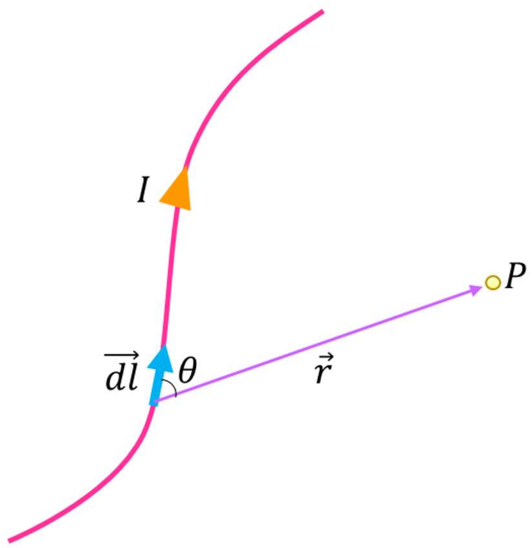
Chapter 4
Moving Charges & magnetism

Biot Savart's law

A current-carrying wire produces a magnetic field around it. Biot-Savart law states that the magnitude of the intensity of small magnetic field dB due to current I carrying element dl at any point P at a distance r from it is:

- $dB \propto I$
- $dB \propto dl$
- $dB \propto \sin\theta$
- $dB \propto \frac{1}{r^2}$



Combining these, we get

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

Or $$\boxed{dB = \frac{\mu_o}{4\pi}\left(\frac{idl\sin\theta}{r^2}\right)}$$

Where θ is the angle between r and dl and $\mu_o = 4\pi \times 10^{-7}\,TmA^{-1}$ is called **magnetic permeability of free space.**

In vector form

$$\boxed{\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{I(dl \times r)}{r^3}}$$

So the **direction of dB is perpendicular to the plane containing** $\vec{r}$ and $\vec{dl}$.

SI unit of magnetic field strength is tesla denoted by T and cgs unit is gauss denoted by 'G' where 1 T = 10^4 G.

Comparison of Coulomb's law and Biot Savart's law

- ✓ Both depend inversely on the square of the distance between the source and point.
- ✓ Magnetic field is produced by a vector source Idl i.e., current element, whereas the electric field is produced by scalar source electric charge q.

- ✓ Electric field is along displacement vector joining source and field point, whereas the magnetic field is perpendicular to the plane containing displacement vector r and current element Idl
- ✓ There is an angle dependence in Biot- Savarts law which is not present in electrostatic case. The magnetic field at any point in the direction of dl is zero.
- ✓ Relation between permeability of free space μ_o to the permittivity ε_o is $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 3 \times 10^8 ms^{-1}$

  where c is the speed of light in vacuum.

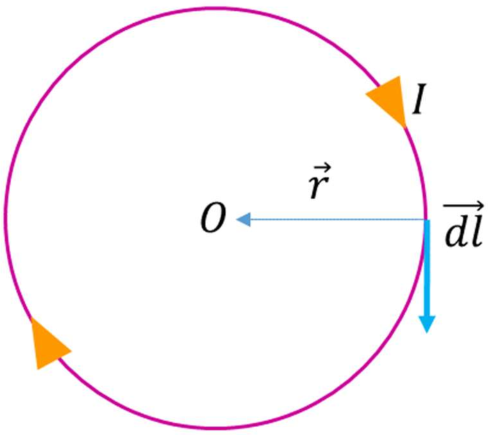
Applications of Biot Savart's law

Magnetic field at the centre of a circular loop carrying current

Consider a circular current carrying loop carrying current I. We have to to find magnetic field at the centre of this loop. Consider a small current element dl on the circumference of this loop. Clearly, angle between dl and r is 90°. Applying Biot Savart's law, we get

$$dB = \frac{\mu_o}{4\pi}\left(\frac{Idl \sin 90^\circ}{r^2}\right)$$

$$\Rightarrow dB = \frac{\mu_o}{4\pi}\frac{Idl}{r^2}$$



Integrating both sides we get

$$\int dB = \int \frac{\mu_o}{4\pi}\frac{Idl}{r^2}$$

$$\Rightarrow B = \frac{\mu_o}{4\pi}\frac{I}{r^2}\int dl$$

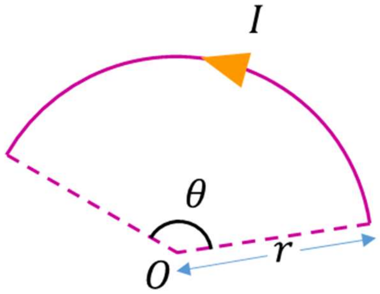
$$\Rightarrow B = \frac{\mu_o}{4\pi}\frac{I}{r^2} \times 2\pi r$$

$$\Rightarrow \boxed{B = \frac{\mu_o I}{2r}}$$

If coil has n turns, then $\boxed{B = \frac{n\mu_o I}{2r}}$

Magnetic field due to arc

As complete circle is also an arc which subtends an angle 2π at the centre so by applying the unitary method, we can find the magnetic field at the centre of arc as follows:

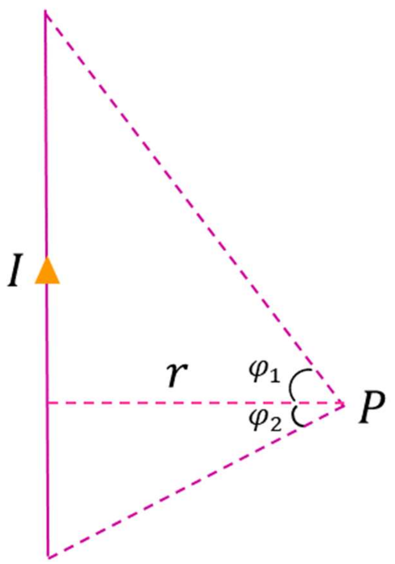


Angle	Magnetic field
2π	$\frac{\mu_o I}{2r}$
1 radian	$\left(\frac{\mu_o I}{2r}\right)\times\frac{1}{2\pi}=\frac{\mu_o I}{4\pi r}$
Any angle θ	$\boxed{B=\frac{\mu_o I}{4\pi r}\times\theta}$

Magnetic field due to a straight conductor

Magnetic field at point P at a perpendicular distance r from from a straight cinductor carrying current I is

$$B=\frac{\mu_o I}{4\pi r}(\sin\varphi_1+\sin\varphi_2)$$



Special cases

When length of wire is infinite (or very long) and distance r is very small then

- ✓ If P lies near one end , then $\varphi_1=90^\circ$ and $\varphi_2=0^\circ$

  so, $B=\frac{\mu_o I}{4\pi r}(\sin 90^\circ+\sin 0^\circ)$

  $\Rightarrow \boxed{B=\frac{\mu_o I}{4\pi r}}$

- ✓ If P lies near centre, then $\varphi_1=90^\circ$ and $\varphi_2=90^\circ$

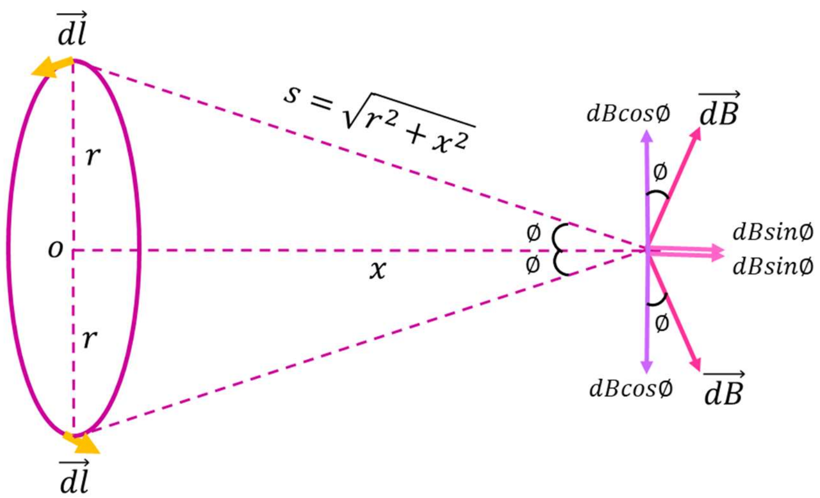
  so, $B=\frac{\mu_o I}{4\pi r}(\sin 90^\circ+\sin 90^\circ)$

  $\Rightarrow \boxed{B=\frac{\mu_o I}{2\pi r}}$

Magnetic field on the axis of a circular loop

Small magnetic field due to current element Idl of circular loop of radius r at a point P at distance x from its centre is

$$dB=\frac{\mu_o}{4\pi}\frac{Idl\sin 90^\circ}{s^2}=\frac{\mu_o}{4\pi}\frac{Idl}{(r^2+x^2)}$$



Component $dB\cos\varphi$ due to current element at point P is cancelled by equal and opposite

component $dB\cos\varphi$ of another diagonally opposite current element, whereas the sine components $dB\sin\varphi$ add up to give net magnetic field along the axis. So net magnetic field at point P due to entire loop is

$$\int dB\sin\varphi = \int_0^{2\pi r} \frac{\mu_o}{4\pi}\frac{Idl}{\left(r^2+x^2\right)}\cdot\frac{r}{\left(r^2+x^2\right)^{1/2}}$$

$$\Rightarrow B = \frac{\mu_o Ir}{4\pi\left(r^2+x^2\right)^{\frac{3}{2}}}\int_0^{2\pi r} dl$$

$$\Rightarrow B = \frac{\mu_o Ir}{4\pi\left(r^2+x^2\right)^{\frac{3}{2}}}.2\pi r$$

$$\Rightarrow \boxed{B = \frac{\mu_o Ir^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}}.$$

Which is directed along the axis (a) towards the loop if current in it is in clockwise direction (b) away from the loop if current in it is in anticlockwise direction.

Special points

- If point P is far away from the centre of the loop i.e. x >> r then magnetic field at point P is

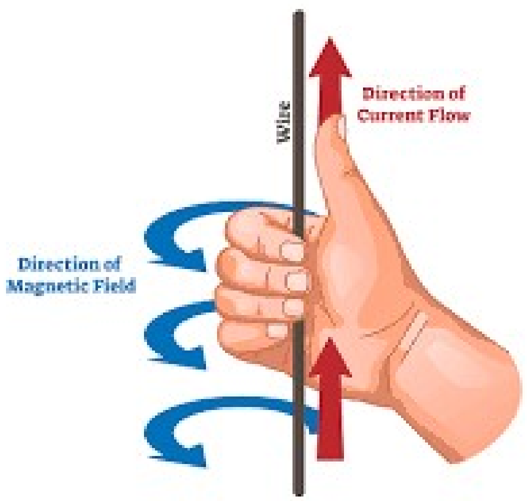
  $$B = \frac{\mu_o Ir^2}{2x} = \frac{\mu_o I\pi r^2}{2\pi x^3} \text{ or } B = \frac{\mu_o IA}{2\pi x^3}$$ where A is the area of the circular loop.

- If circular loop has N turns then magnetic field strength at its centre is $B = \frac{\mu_o NI}{2r}$ and at any point on the axis of circular loop is $B = \frac{\mu_o NIr^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}$

Direction of magnetic field

Right hand thumb rule or right hand grip rule or right hand palm rule

It states that if we hold a current I carrying wire in our right hand, such that the thumb points in the direction of current, then the curled fingers around it give us the direction of magnetic field lines around it.



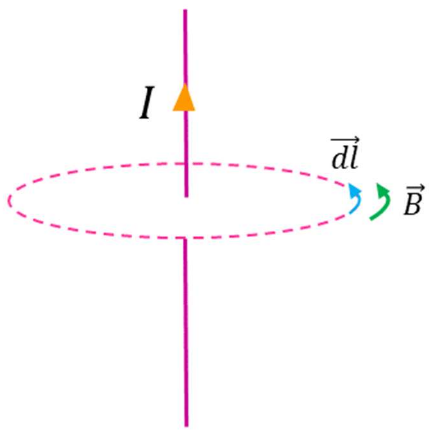
Ampere's circuital law

It states that the line integral of magnetic field intensity over a closed loop is μ_o times the total current threading the loop.

$$\int \vec{B}.\vec{dl} = \mu_o I$$

Proof:

Consider a straight conductor carrying current as shown in the figure. Consider a circular Amperian loop of radius r around the conductor. As $\vec{B}$ and $\vec{dl}$ are in same direction so angle between them is 0. Therefore



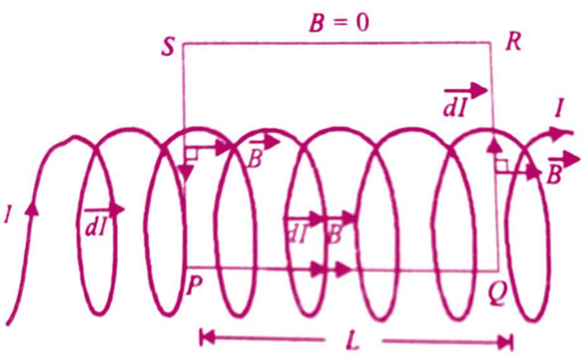
$$\int \vec{B}.\vec{dl}$$
$$= \int Bdl\cos 0^\circ$$
$$= \int Bdl$$
$$= B\int dl$$
$$= \frac{\mu_o I}{2\pi r} \times 2\pi r$$
$$= \mu_o I$$
$$\therefore \int \vec{B}.\vec{dl} = \mu_o I$$

Applications of ampere's circuital law

Magnetic field intensity at the centre of a long solenoid

Let a solenoid consists of n no. of turns per unit length and carry current I. Then magnetic field lines inside the solenoid are parallel to its axis whereas outside the solenoid the magnetic field is zero. Line integral of magnetic field over a closed loop PQRS shown in the figure is

$$\int \vec{B}.\vec{dl} = \int_P^Q \vec{B}.\vec{dl} + \int_Q^R \vec{B}.\vec{dl} + \int_R^S \vec{B}.\vec{dl} + \int_S^P \vec{B}.\vec{dl}$$



$$= \int_P^Q B.dl\cos 0^\circ + \int_Q^R B.dl\cos 90^\circ + 0 + \int_S^P B.dl\cos 90^\circ$$
$$= B\int_P^Q dl + 0 + 0 + 0 = BL$$

But by Ampere's circuital law

$$\int \vec{B}.\vec{dl} = \mu_o \times \text{total current threading loop PQRS}$$
$$= \mu_o \times \text{number of turns in solenoid PQRS} \times I$$
$$= \mu_o nLI$$

Therefore

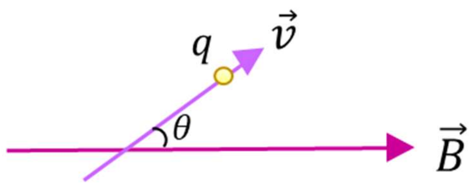
$$BL = \mu_o nLI \Rightarrow \boxed{B = \mu_o nI}$$

Note: at the ends of the solenoid the magnetic field is $\boxed{B = \frac{1}{2}\mu_o nI}$

Force acting on a charged particle moving in a magnetic field

If a charge q is moving with velocity v in a magnetic field of intensity B such that the angle between velocity vector and magnetic field vector is θ, then a force F acts on the particle such that

i) $F \propto q$
ii) $F \propto v$
iii) $F \propto B$
iv) $F \propto \sin\theta$



Combining all these, we get

$F \propto qvB\sin\theta$
$\Rightarrow \boxed{F = qvB\sin\theta}$

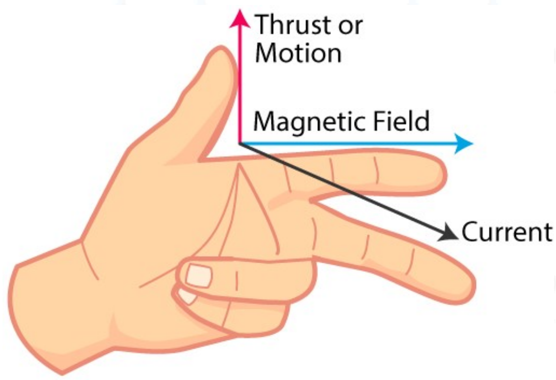
As the value of constant in this relation is 1 in SI units.

In vector form

$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, thus F is perpendicular to the plane containing v and B.

Direction of F can be found by Fleming's left hand rule

It states that stretch the thumb, forefinger, and central finger of the left hand in a mutually perpendicular position such that the forefinger is pointing towards the direction of the magnetic field, central finger pointing towards the direction of motion of positive charge (direction of current) then the direction of thumb gives the direction of force acting on the particle.



Definition of 1 tesla

Since $B = \frac{F}{qv\sin\theta}$

So If $q = 1\,C,\ v = 1\ ms^{-1}, \theta = 90^\circ\ (\sin 90^\circ = 1)$, then $B = 1T$

Magnetic field is said to be 1 tesla when a charge of 1 coulomb moving at a speed of 1 m/s perpendicularly to the direction of field experiences a force of 1 newton in it.

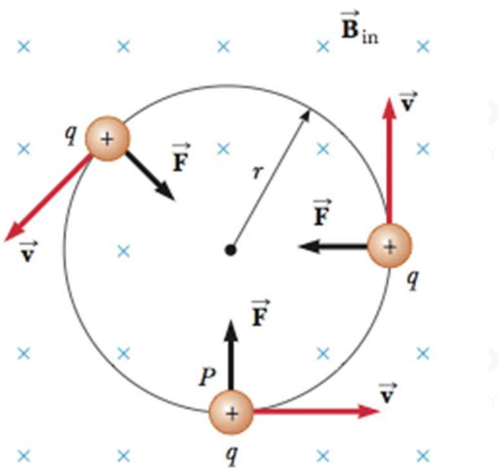
If a charge q enters perpendicularly into a magnetic field, then its path will be circular as force always acts in a direction perpendicular to the direction of motion of motion of the charge. Centripetal force required for circular motion is provided by the magnetic force acting on the particle. Thus

$$\frac{mv^{\not 2}}{r} = q\not vB$$

$$\frac{mv}{r} = qB$$

Radius of the path (r)

$$r = \frac{mv}{Bq}$$



Velocity (v)

$$v = \frac{Bqr}{m}$$

Time period (T)

$$T = \frac{2\pi r}{v} = \frac{2\pi \not r}{\frac{Bq\not r}{m}} = \frac{2\pi m}{Bq}$$

Frequency

$$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

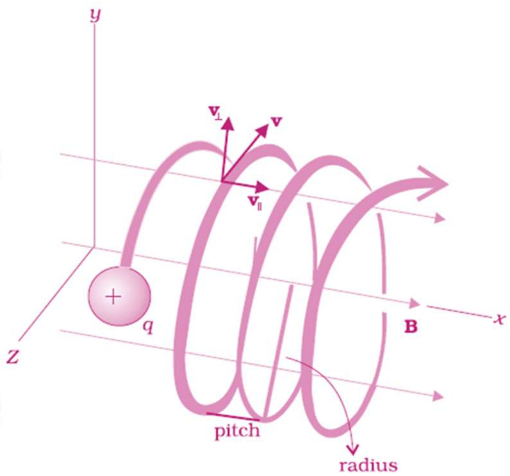
Angular frequency

$$\omega = 2\pi\nu = 2\pi \times \frac{Bq}{2\pi m} = \frac{Bq}{m}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Bqr}{m}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}\not m\frac{B^2q^2r^2}{m^{\not 2}} = \frac{1}{2}\frac{B^2q^rr^2}{m}$$



If charge particle enters at an angle with the direction of magnetic field then split its velocity into rectangular components $v\cos\theta$ along the field and $v\sin\theta$ perpendicular the field as shown. Due to these two components, the motion of the charge is helical. Distance between two turns of the helix is called **pitch(d)** which is given by

$$d = v\cos\theta \times \text{time period} = v\cos\theta \times \frac{2\pi m}{Bq}$$

Lorentz force

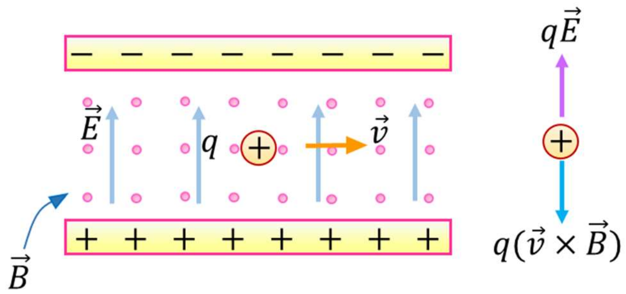
Force acting on a particle in a region where both electric and magnetic fields exist is called Lorentz force. Lorentz force is the resultant of electric and magnetic force acting on the particle.

$$F = \vec{F}_E + \vec{F}_B$$

$$F = q\vec{E} + q\left(\vec{v} \times \vec{B}\right)$$

Velocity selector or velocity filter

Consider a situation as shown in the figure in a charge is moving perpendicularly to both electric and magnetic fields such the force the force acting on charge due to both the fields is equal and opposite i.e.



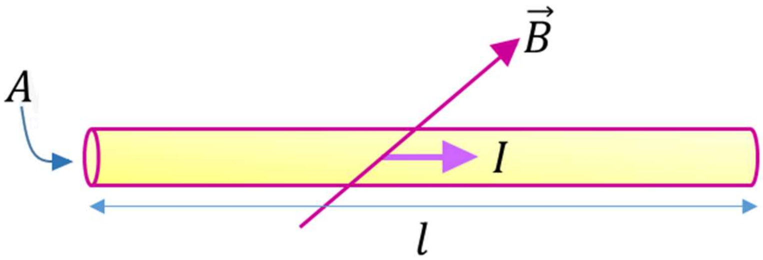
qE = qvB

$$\therefore v = \frac{E}{B}$$

This result is used in velocity selectors or velocity filters in which we have to select a particle with a particular value of velocity.

Force acting on a current carrying conductor placed in a magnetic field

Consider a conductor of length ℓ and area of cross section A carrying current I placed in a magnetic field at an angle θ as shown. If number density of electrons in the conductor is n then total number of electrons in the conductor is $A\ell n$.



As force acting on one electron is $f = ev_d B\sin\theta$ where v_d is the drift velocity of electrons.

So the total force acting on the conductor is

$$A\ell nf = A\ell n\left(ev_d B\sin\theta\right)$$

$$= \left(Anev_d\right)\ell B\sin\theta$$

$$\Rightarrow \boxed{F = I\ell B\sin\theta}$$

Direction of this force can be determined by Fleming's left hand rule.

Force between two parallel straight conductors carrying current

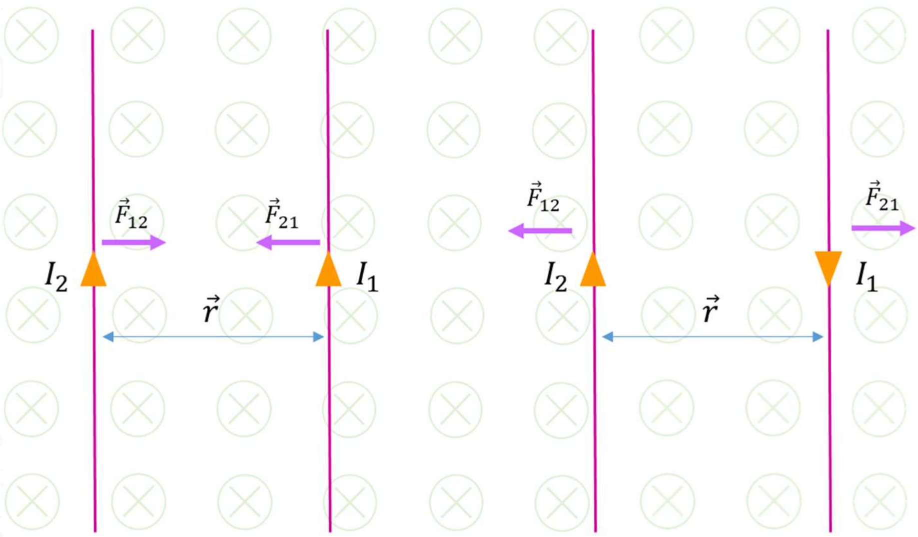
When the currents are in same direction

When two current carrying conductors are placed parallel to each other, each conductor produces a magnetic field around itself. So, one conductor is placed in the magnetic field produced by the other. Using Fleming's left hand rule it can be easily shown that the forces on them are such that they attract each other. Force acting on 1st conductor is given as

$$F_1 = I_1 \ell B_2 \sin 90^\circ$$

$$F_1 = I_1 \ell \frac{\mu_o I_2}{2\pi r}$$

$$\Rightarrow \boxed{\frac{F_1}{\ell} = \frac{\mu_o I_1 I_2}{2\pi r}}$$



Now force acting on conductor 2 is given by

$$F_2 = I_2 \ell B_1 \sin 90^\circ$$

$$F_2 = I_2 \ell \frac{\mu_o I_1}{2\pi r}$$

$$\Rightarrow \boxed{\frac{F_2}{\ell} = \frac{\mu_o I_1 I_2}{2\pi r}}$$

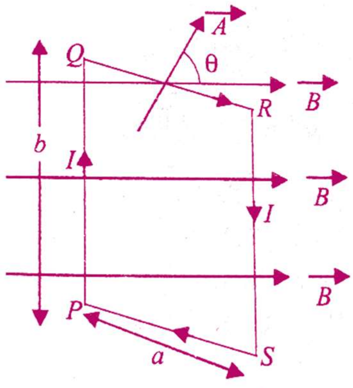
Therefore $F_1 = F_2$

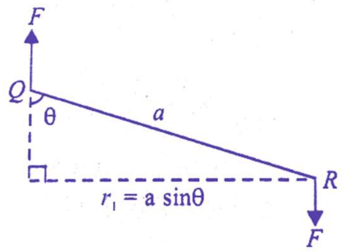
When the current is in opposite direction

the conductors will repel each other the magnitude of force will be same as derived above.

Torque acting on a current carrying conductor placed in a magnetic field

When a rectangular loop PQRS of sides 'a' and 'b' carrying current I is placed in uniform magentic field B, such that area vector A makes an angle θ with direction of magnetic field, then forces on the arms QR and SP of loop are equal, opposite and collinear, thereby perfectly cancel each other, whereas forces on arms PQ and RS of loop are equal and opposite but not collinear, so they give rise to torque on loop.





Force on side PQ or RS of loop is $F = IbB\sin 90^\circ = IbB$

Perpendicular distance between two non collinear forces $r_\perp = a\sin\theta$

So, torque on the loop is

$\tau = F_\perp = IbBa\sin\theta = I(ab)B\sin\theta$

or $\boxed{\tau = IAB\sin\theta}$

If loop has N turns then $\boxed{\tau = NIAB\sin\theta}$.

In vector form $\tau = \vec{M}\times\vec{B}$ where M = NIA is called magnetic dipole moment of current loop abd is directed in direction of area vector.

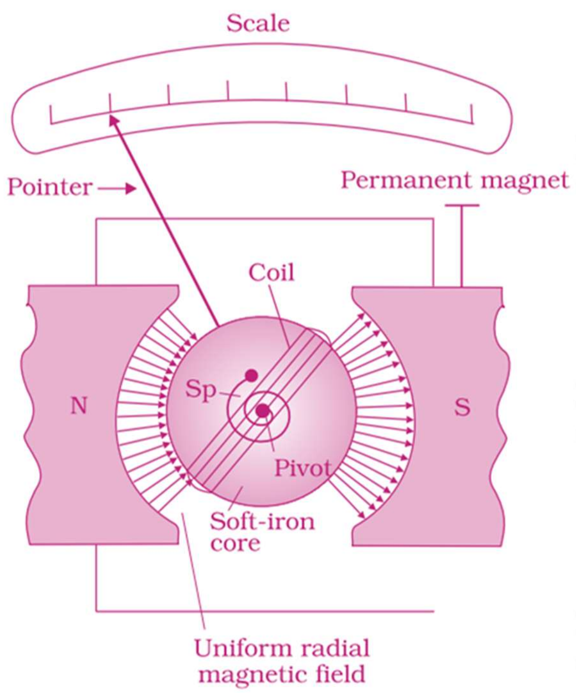
- ✓ If the plane of the loop is normal to the direction of magnetic field i.e. θ = 0° between $\vec{B}$ and $\vec{A}$ then the loop does not experience any torque i.e. $\tau_{min} = 0$
- ✓ If the plane of the loop is parallel to the direction of magnetic field i.e. θ = 90° between $\vec{B}$ and $\vec{A}$ then the loop experience maximum torque $\tau_{max} = NIAB$

Moving coil galvanometer

Moving coil galvanometer is used to detect or measure small currents. It works on the principle that when an electric current is passed through a coil placed in a magnetic field, it experiences a torque, whose magnitude is proportional to the strength of electric current passed through it.

In order to make torque on the coil independent of angle θ between the area vector A and magnetic field B, so that the plane of coil always remain parallel to the field.

1) The radial magnetic field is applied by cutting the poles of magnet concave, and
2) The core of coil is filled by soft iron



Therefore, when current is passed through a coil suspended in radial magnetic field, it experiences a torque NIAB and gets deflected by an angle θ where it is balanced by restoring torque kθ, developed in suspension strip, where k is restoring torque per unit deflection or torsional constant of suspension strip.

Thus NIAB = kθ

Or $I = \frac{k}{NAB}\theta$

$I = G\theta$

or $I \propto \theta$

So by measuring deflection α, we can measure current I passing through the coil.

Where $G = \frac{k}{NAB}$, G is called galvanometer constant.

So measuring by deflection α, we can measure current I passing through the coil.

Current sensitivity (I_s)

It is defined as the deflection produced in the galvanometer coil when unit current is passed through it.

Thus $I_s = \frac{\theta}{I}$. SI unit is rad/A.

since $\frac{\theta}{I} = \frac{1}{G} = \frac{NAB}{k}$ therefore, the increase current sensitivity we should

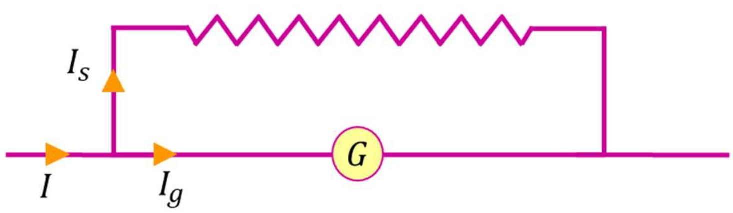
- ✓ Increase N which is not possible beyond a certain limit as it makes galvanometer bulky.
- ✓ Increase A which is not possible beyond a certain limit due to space.
- ✓ Increase B
- ✓ Decrease k, so we use phosphor bronze strip in galvanometer because it has very small k.

Voltage sensitivity (V_s)

Is the defined as the deflection produced in galvanometer coil when unit voltage is applied across its terminals. $V_s = \frac{\theta}{V}$. SI unit is rad/V.

Conversion of galvanometer into ammeter

A galvanometer can be converted into ammeter by connecting a low shunt resistance in parallel with it, so that most of the current by passes through the shunt resistance, enabling the galvanometer to measure much larger currents.



Thus if a galvanometer of resistance R_g which gives full scale deflection at I_g is to be used to convert into an ammeter capable of measuring a maximum current I , we connect a shunt resistance R in parallel with it which is obtained as

$$V_R = V_G \Rightarrow (I - I_g)R = I_g R_g$$

$$\Rightarrow \boxed{R = \frac{I_g R_g}{I - I_g}}$$

Conversion of galvanometer into voltmeter

A galvanometer can be converted into voltmeter by connecting high resistance in series with it, so that most of the voltage applied drops across it, enabling the galvanometer to measure much larger voltages.

Thus is the galvanometer of resistance R_g which gives full deflection at current I_g, is to be converted into voltmeter capable of measuring maximum voltage up to V volts, then a high resistance R is connected in series with it which is given by

$$V = I_g R_g + I_g R \text{ or } V - I_g R_g = I_g R \text{ or } \boxed{R = \frac{V}{I_g} - R_g}$$

Figure of merit

The figure of merit of a galvanometer is a measure that indicates its sensitivity. It is defined as the current required to produce a deflection of one scale division in the galvanometer. In other words, it tells you how much current is needed to achieve a certain amount of movement in the needle or indicator of the galvanometer.

Mathematically, the figure of merit (K) of a galvanometer can be expressed as:

$$\boxed{k = \frac{I_g}{n}}$$

- SI unit of k is division per ampere.