Chapter 7
Alternating Current

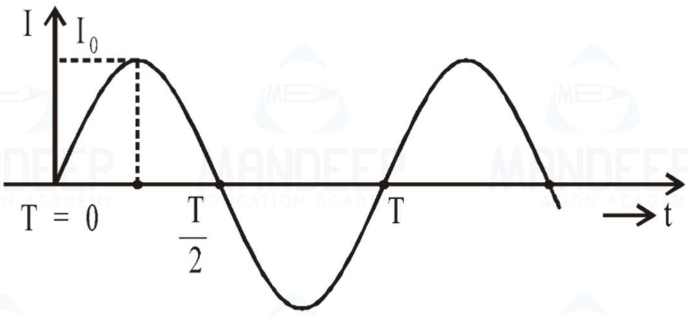
Alternating current

Definition: Alternating Current is a type of electrical current where the flow of electric charge periodically reverses direction. In contrast to direct current (DC), where the flow of electric charge is only in one direction, AC changes direction at a regular interval.

Frequency: This is the number of times the current changes direction per second, measured in Hertz (Hz). For example, in many countries, the standard power supply frequency is 50 Hz or 60 Hz.

Waveform: In most AC systems, the waveform is sinusoidal. However, it can also be triangular or square in some applications.

Equation of AC is $I = I_0 \sin\omega t$ or $I = I_0 \cos\omega t$. Graph of AC is shown in figure below:



Applications: AC is commonly used for power distribution in homes and industries because it can be easily transformed to different voltages using transformers. This property makes it more efficient for long-distance power transmission.

Alternating Voltage (AC Voltage)

Definition: Alternating Voltage is the voltage in an electrical circuit where the magnitude and direction of the voltage vary cyclically, as opposed to Direct Voltage (DC), where the direction of the voltage remains constant.

Amplitude: This refers to the maximum value of the voltage. For AC voltage, this is often described as peak voltage (Vp) or root mean square voltage (Vrms). Vrms is particularly useful as it provides a comparable value to DC voltage in terms of the power delivered.

Phase: In AC circuits, voltage can be out of phase with the current. This phase difference is important in understanding how AC circuits operate, especially in the context of power factor and reactive loads.

Waveform: Like AC current, the most common waveform for AC voltage is sinusoidal, but it can also take other forms depending on the application.

Alternating EMF equation is $E = E_o \sin\omega t$ or $E = E_o \cos\omega t$. Graphical variation of E is same as that of I.

Root mean square (RMS) or virtual value of AC

The instantaneous power dissipated in the resistor is

$$P = i^2R = i_m^2 R \sin^2 \omega t$$

The average value of p over a cycle is

$$\bar{p} = \langle i^2R \rangle = \langle i_m^2 R \sin^2 \omega t \rangle$$
$$\Rightarrow \bar{p} = i_m^2 R \langle \sin^2 \omega t \rangle$$
$$\because \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

Thus,

$$\bar{p} = \frac{1}{2} i_m^2 R$$

Now equating this to dc power $P = I^2R$, we get

$$I = \sqrt{\frac{1}{2} i_m^2} = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

Thus, $\boxed{I_{rms} = 0.707 i_m}$

Therefore,

$$\boxed{I_v \text{ or } I_{rms} \text{ or } I_{eff} = \frac{I_0}{\sqrt{2}} = 0.707 I_0}$$

$$\boxed{V_v \text{ or } V_{rms} \text{ or } V_{eff} = \frac{V_m}{\sqrt{2}} = 0.707 V_m}$$

The R.M.S value of AC is 70.7% times its peak value and it remains the same even for a complete cycle.

Average EMF & current:

$$\boxed{I_{av} = I_m \times \frac{2}{\pi} = 0.637 I_m}$$

$$\boxed{V_{av} = V_m \times \frac{2}{\pi} = 0.637 V_m}$$

Phasor diagram

A phasor is a vector which rotates about the origin with angular speed ω

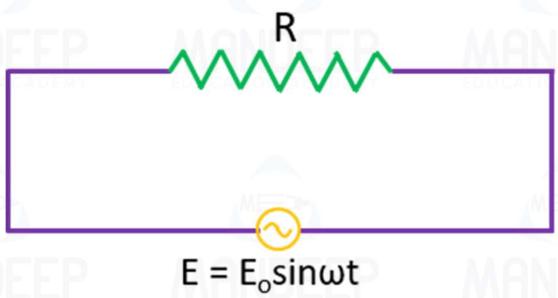
In an AC circuit, the EMF and current vary sinusoidally with time and is represented as,

$$V = V_m \sin\omega t$$
$$I = I_m \sin(\omega t + \varphi)$$

Where φ is the phase angle between alternating EMF and current.

AC circuit containing resistor only

Consider a resistor of resistance R connected to an alternating emf source as shown.



Let the applied emf be

$E = E_o \sin \omega t$.

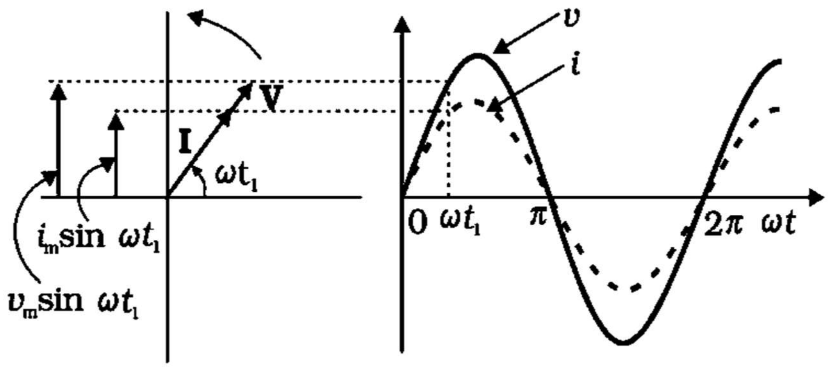
Dividing both sides by R, we get

$$\frac{E}{R} = \frac{E_o}{R} \sin \omega t$$

$$\frac{E}{R} = \frac{E_o}{R} \sin \omega t$$

$$\Rightarrow \boxed{I = I_o \sin \omega t}$$

Therefore, current and voltage are in same phase. Graph and phasor diagram is shown below:



AC circuit containing inductor only

Consider an inductor of inductance L connected to an AC source as shown. Let the applied emf be

$E = E_o \sin \omega t$.

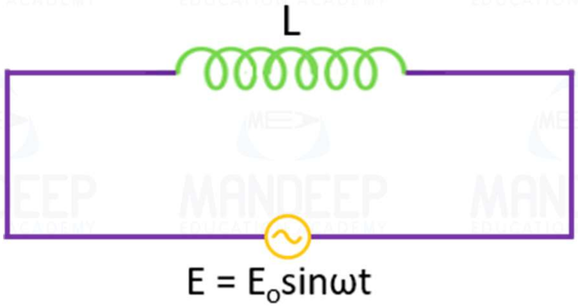
Since $E = L\frac{dI}{dt}$

Therefore $dI = \frac{E}{L} dt$

$$\Rightarrow dI = \frac{E_o \sin\omega t}{L} dt$$

Integrating both sides we get

$$\Rightarrow \int dI = \int \frac{E_o}{L} \sin\omega t dt$$



$$\Rightarrow I = \frac{E_o}{L} \int \sin\omega t dt$$

$$\Rightarrow I = \frac{E_o}{\omega L}[-\cos\omega t]$$

$$\because \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\therefore I = -\frac{E_o}{\omega L}\left[\sin\left(\frac{\pi}{2} - \omega t\right)\right]$$

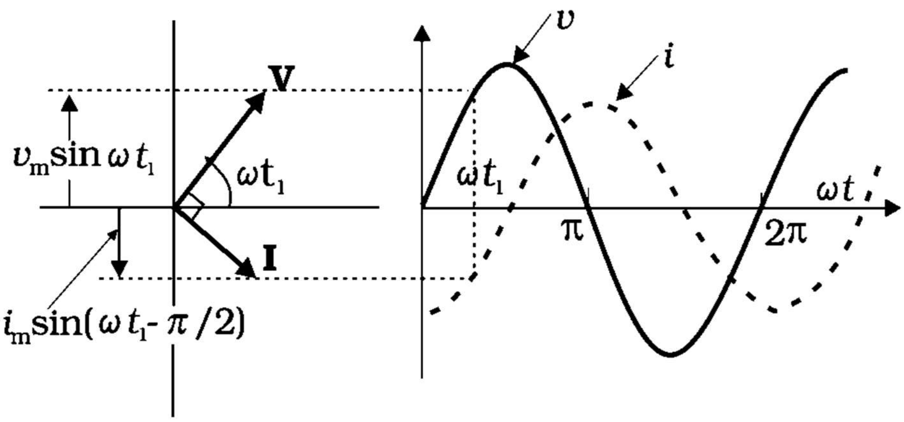
As $\omega L = X_L$

$$\Rightarrow I = \frac{E_o}{X_L}\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$\Rightarrow \boxed{I = I_o\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]}$$

where $I_o = \frac{E_o}{X_L}$

Thus, current lags behind voltage in a purely inductive circuit. Graph and phasor diagram is shown:

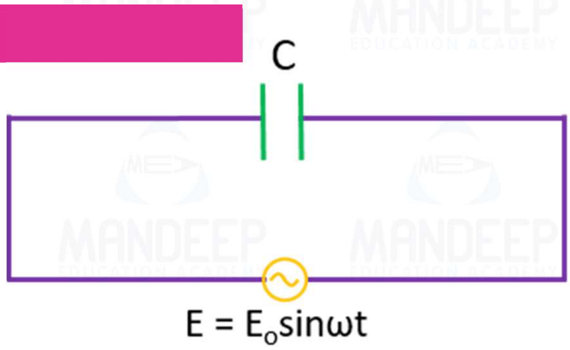


AC circuit containing capacitor only

Consider an inductor of inductance L connected to an AC source as shown. Let the applied emf be

$E = E_o \sin\omega t$.

Since Q = CE



$Q = CE_o \sin\omega t$

$Q = CE_o \sin\omega t$

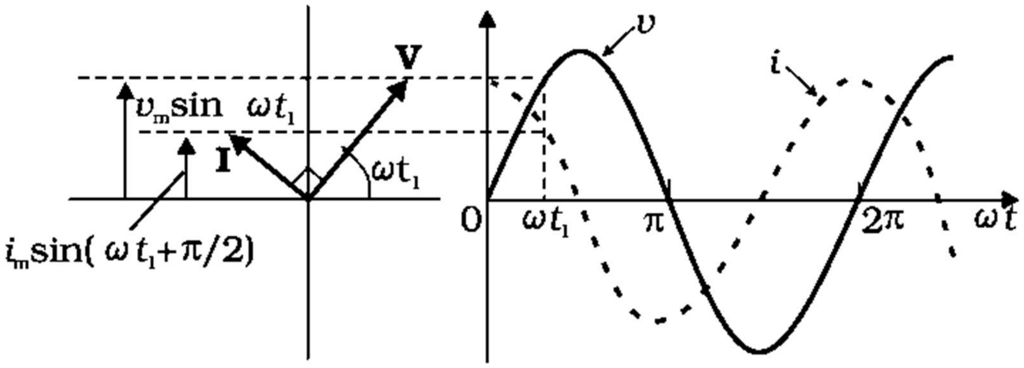
$$I = \frac{dQ}{dt} = \frac{d}{dt}[CE_o \sin\omega t]$$

$$\Rightarrow I = \omega CE_o \cos\omega t$$

$$\Rightarrow I = \frac{E_o}{\frac{1}{\omega C}} \cos\omega t$$

$$\Rightarrow \boxed{I = I_o \sin(\omega t + \frac{\pi}{2})}$$

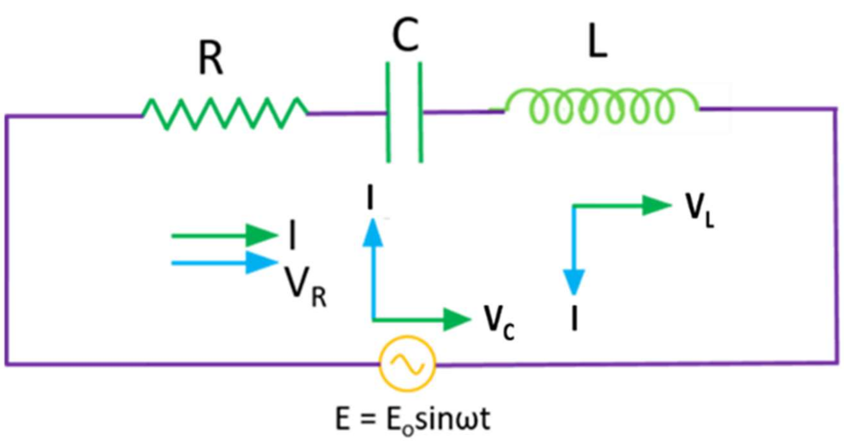
Where $\frac{E_o}{\frac{1}{\omega C}} = I_o$. Thus, current leads the voltage by a phase of $\frac{\pi}{2}$ in a purely capacitive circuit. Graph and phasor diagram is shown below



Impedance in series LCR circuit

Consider a resistor of resistance R, inductor of inductance L and capacitor of capacitance C connected in series to an alternating EMF source as shown.
Voltage across all the components is shown in the diagram below. Net voltage V is given by

$$V = \sqrt{(V_L - V_C)^2 + V_R^2}$$



$\because V_L = IX_L,\ V_R = IR,\ V_C = IX_C$

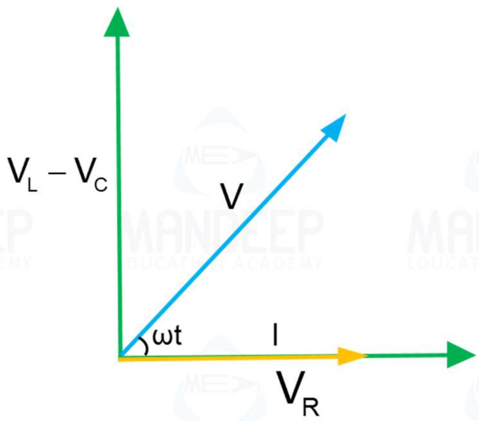
$\therefore V = \sqrt{(IX_L - IX_C)^2 + (IR)^2}$

$\Rightarrow V = \sqrt{I^2\{(X_L - X_C)^2 + R^2\}}$

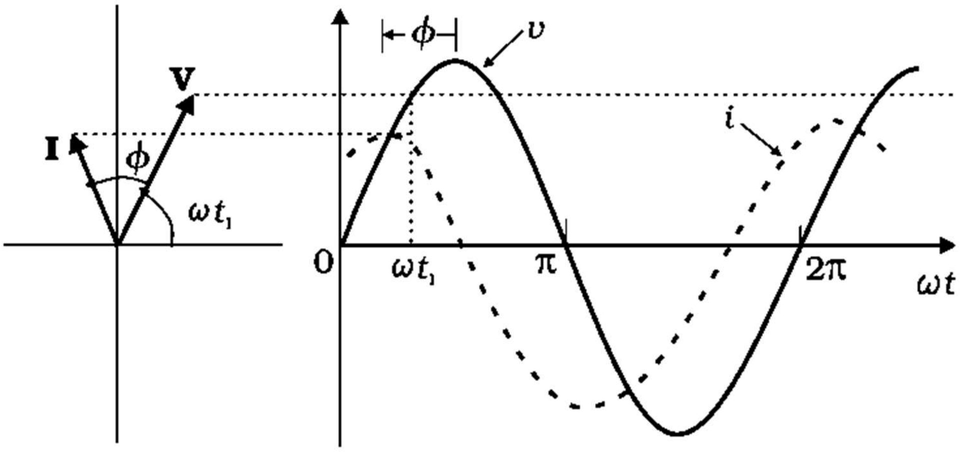
$\Rightarrow V = I\sqrt{\{(X_L - X_C)^2 + R^2\}}$

$\Rightarrow \dfrac{V}{I} = \sqrt{\{(X_L - X_C)^2 + R^2\}}$

$$\Rightarrow \boxed{Z = \sqrt{(X_L - X_C)^2 + R^2}}$$

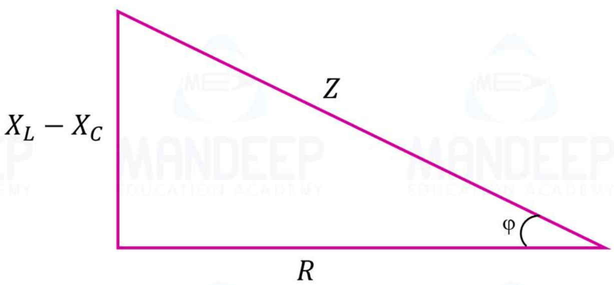


Where Z is called the impedance of the circuit. Phasor diagram and graph for series LCR circuit is shown in the figure below:



Impedance triangle

All formulae of series LCR circuit can be obtained using a right angled triangle (called impedance triangle) as shown below: If we apply Pythagoras theorem on this triangles, we get



$Z^2 = (X_L - X_C)^2 + R^2$

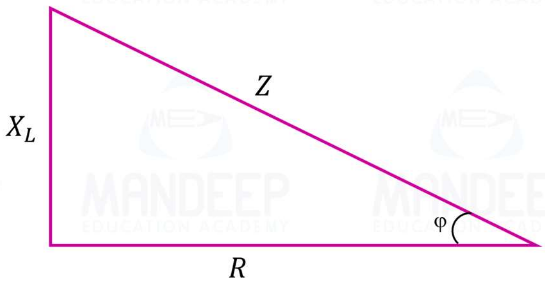
$\Rightarrow Z = \sqrt{(X_L - X_C)^2 + R^2}$

Which is formula for impedance.

Also, $\tan\varphi = \dfrac{X_L - X_C}{R}$, $\cos\varphi = \dfrac{R}{Z}$.

Series LR circuit

Impedance of circuit is given by



$Z = \sqrt{X_L^2 + R^2}$

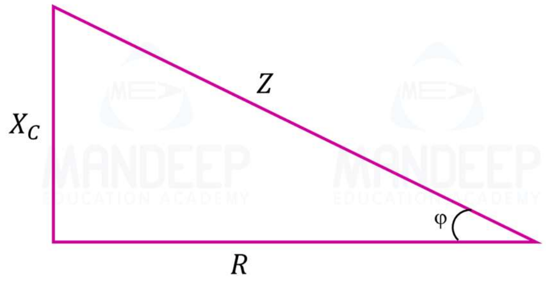
Phase difference is given by : $\cos\varphi = \frac{R}{Z}$, $\tan\varphi = \frac{X_L}{R}$

Series CR circuit

$Z = \sqrt{X_C^2 + R^2}$

Phase difference is given by :

$\cos\varphi = \frac{R}{Z}$, $\tan\varphi = \frac{X_C}{R}$



Resonating frequency in series LCR circuit

Resonance occurs when inductive reactance becomes equal to capacitive reactance. At this frequency the impedance of the circuit becomes minimum and current becomes maximum.

$X_L = X_C$

$\Rightarrow 2\pi v_r L = \frac{1}{2\pi v_r C}$

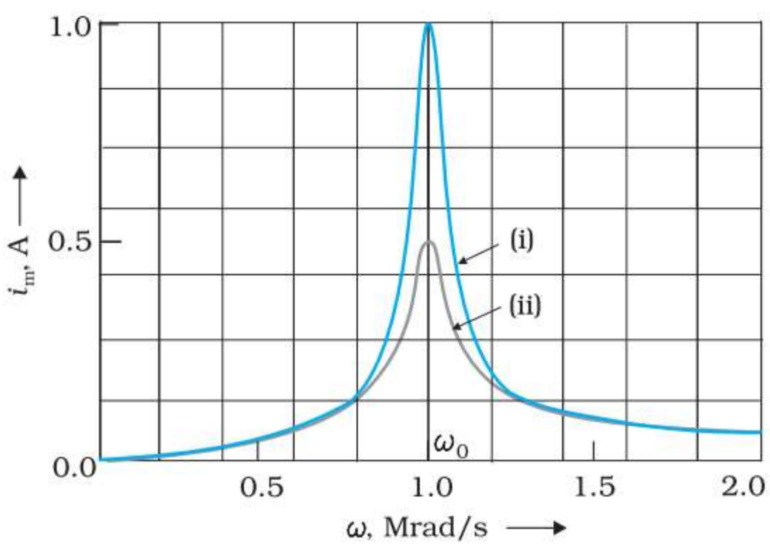
$\Rightarrow (2\pi v_r)^2 = \frac{1}{LC}$

$\Rightarrow 2\pi v_r = \frac{1}{\sqrt{LC}}$

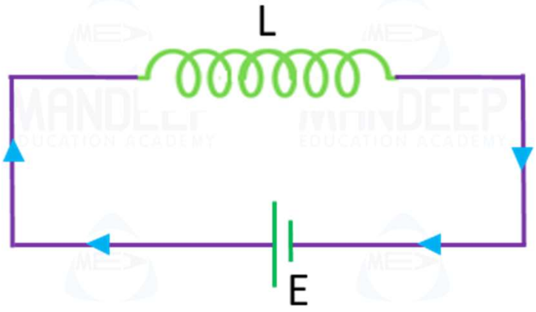
$\Rightarrow \boxed{v_r = \frac{1}{2\pi\sqrt{LC}}}$,

v_r is called resonance frequency.

Graph between current and angular frequency is shown below:



Energy stored in an inductor



Consider an inductor of inductance L connected to a voltage source E as shown in figure above. Let current at any instant be I.

As we know that instantaneous power is given by

$$P = EI$$

$$\text{As } E = L\frac{dI}{dt}$$

$$\text{so, } P = LI\frac{dI}{dt}$$

$$\because P = \frac{dW}{dt}$$

$$\therefore \frac{dW}{dt} = LI\frac{dI}{dt}$$

$$\Rightarrow dW = LIdI$$

So, total work done by source to build a maximum current I_o in the circuit is

$$\Rightarrow W = \int_0^{I_o} LIdI$$

$$\Rightarrow W = L\left[\frac{I^2}{2}\right]_0^{I_o}$$

$$\Rightarrow W = L\left[\frac{I_0^2}{2} - 0\right]$$

$$\Rightarrow W = \frac{1}{2}LI_o^2$$

This work is stored in the circuit as magnetic potential energy. So,

$$\boxed{U_B = \frac{1}{2}LI_o^2}$$

Power in series LCR circuit

Let a voltage $E = E_o \sin\omega t$ be applied to a series LCR circuit and current flowing through it is $I = I_o \sin\omega t$, so instantaneous power supplied to the source is

$$P = EI = E_o \sin\omega t \times I_o \sin(\omega t - \varphi)$$
$$= \frac{E_o I_o}{2}\left[\cos\varphi - \cos(2\omega t + \varphi)\right]$$

The average power over a cycle is given by the average of the two terms in RHS of above equation. It is only the second term which is time dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore

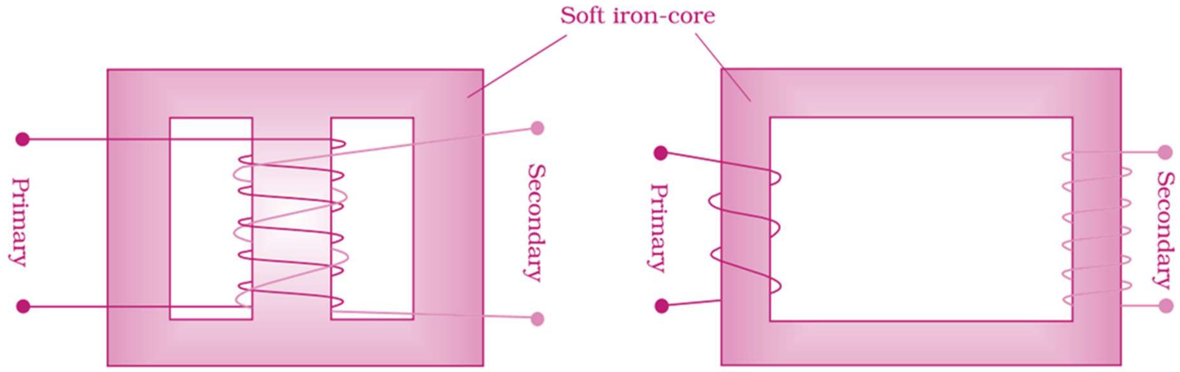
$$P = \frac{E_o I_o}{2}\cos\varphi = \frac{E_o}{\sqrt{2}}\frac{I_o}{\sqrt{2}}\cos\varphi$$
$$\Rightarrow \boxed{P = E_{rms} I_{rms} \cos\varphi}$$

For purely inductive or purely capacitive circuit.

Transformer

TRANSFORMER

A device used for converting high current low alternating voltage into low current, high voltage and vice versa.



Working principle:

Transformer works on the principle of mutual induction i.e., if two coils are inductively coupled, then a change in current in one coil induces an EMF in another.

Construction:

- **Step up transformer**

  In these transformers, number of turns in the secondary coils is greater than primary ones i.e., ($N_S > N_P$). The primary coil is made of thick insulated wire. It helps in converting 'mow voltage/high current' to 'high voltage/low current'.

- **Step down transformer:**

In such transformer, number of turns in the secondary coil are less than in the primary coils i.e., ($N_S < N_P$). The primary coils are made of thin wire and secondary coils are made of thick wires. It converts 'high voltage/low current' into 'low voltage/high current'.

Theory:

When current is made to pass through the primary coils, a magnetic field is produced which in turn links to the secondary coils. Hence, an induced EMF is produced in secondary coils due to change of flux. Let the magnetic flux with primary and secondary coils at any instant be ϕ_P and ϕ_S then,

$$\frac{\phi_S}{\phi_P} = \frac{N_S}{N_P} \Leftrightarrow \phi_S = \frac{N_S}{N_P}\phi_P$$

Differentiating w.r.t time, $\frac{d\phi_S}{dt} = \frac{N_S}{N_P}\frac{d\phi_P}{dt}$

From Faraday's law of electromagnetic induction, an induced EMF is produced with the change in flux and is given by,

$$\varepsilon = -\frac{d\phi}{dt}$$

If E_P and E_S be the induced EMF in the primary and secondary coils respectively, then

$$E_P = -\frac{d\phi_P}{dt}; E_P = -\frac{d\phi_S}{dt}$$

Hence, $E_S = \frac{N_S}{N_P}E_P$

$$\frac{E_S}{E_P} = \frac{N_S}{N_P}$$

The ratio $\frac{N_S}{N_P} = K$ is known as **transformation ratio.** Assuming that there is no power loss in the transformer, instantaneous input power = instantaneous output power

$$\frac{d\phi_S}{dt} = \frac{N_S}{N_P}\frac{d\phi_P}{dt}$$

$$I_P E_P = I_S E_S$$

$$\boxed{\frac{E_S}{E_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}}$$

ENERGY LOSS IN A TRANSFORMER

- **Flux loss:** The flux generated in the primary coil does not completely link to the secondary coils.
- **Copper loss:** Since, the length of the coils is large, the resistance of the coils leads to wastage of energy due to Joule's heating effect.

- **Iron loss:** The change in magnetic flux produces eddy currents in the iron core, which leads to wastage of energy in the form of heat. Use of laminated iron core minimizes this effect.
- **Hysteresis loss:** The AC Current flowing through the coil magnetizes the core periodically. As a result of which energy is lost due to hysteresis. This loss is minimized by appropriate selection of material of core, which has narrow hysteresis loss.
- **Humming loss:** Due to AC current, transformer produces humming sound, causing further loss of energy.

Efficiency:

Hence, the output power (P_S) is less than the input power (P_P) i.e., $I_P E_P > I_S E_S$. The efficiency of the transformer is

$$\eta = \frac{I_S E_S}{I_P E_P}$$