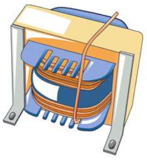


Chapter 6
Electromagnetic Induction

Magnetic flux

The magnetic flux (φ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing a closed area (A). If θ is the angle between the direction of the field and normal to the area, Then

$$\phi = \vec{B}\cdot\vec{A}$$

$$\text{or } \phi = BA\cos\theta$$

SI unit of magnetic flux is weber (Wb).

$$\boxed{1\text{ Wb} = 1\text{ Tm}^2}$$

Therefore, unit of magnetic field intensity is also called Wb/m^2

magnetic flux through an area of 1 m^2 is said to be 1 weber when the area is kept in a magnetic field of 1 tesla such that the angle between area vector and magnetic field vector is 0º.

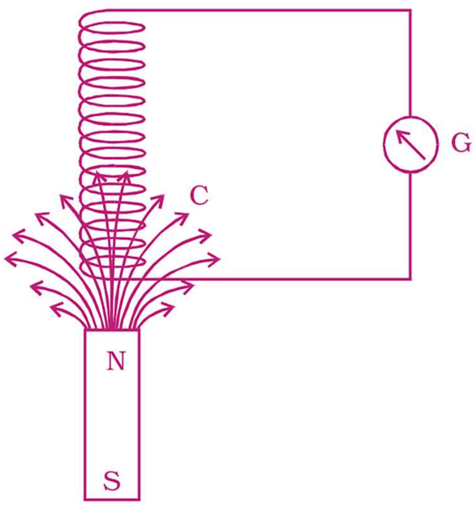
Electromagnetic induction

Whenever there is a change in the magnetic flux linked with a closed circuit an emf is produced. This emf is known as the induced emf and the current that flows in the closed circuit is called induced current. **The phenomenon of producing an induced emf due to the changes in the magnetic flux associated with a closed circuit is known as electromagnetic induction.**

Faraday's experiment and results

In the year 1820, Hans Christian Oersted demonstrated that a current carrying conductor is associated with a magnetic field.

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction.



He made the following inferences:

1) Whenever there is a relative motion between the coil and the magnet, the galvanometer shows deflection indicating the flow of induced current.
2) The deflection is momentary. It lasts so long as there is relative motion between the coil and the magnet.
3) The direction of the flow of current changes if the magnet is moved towards and withdrawn from it.
4) The deflection is more when the magnet is moved faster, and less when the magnet is moved slowly.

5) However, on reversing the magnet (i.e.) south Pole pointing towards the coil, same results are obtained, but current flows in the opposite direction.

Faraday's laws of electromagnetic induction

Based on his studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

First law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second law

The magnitude of emf induced in a closed circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Let ϕ_1 be the magnetic flux linked with the coil initially and ϕ_2 be the magnetic flux linked with the coil after a time t. Then Rate of change of magnetic flux = $\frac{\phi_2 - \phi_1}{t}$

According to Faraday's second law, the magnitude of induced emf is, $\varepsilon \propto \frac{\phi_2 - \phi_1}{t}$

If $d\phi$ is the change in magnetic flux in a time dt, then the above equation can be written as $\varepsilon \propto \frac{d\phi}{dt}$

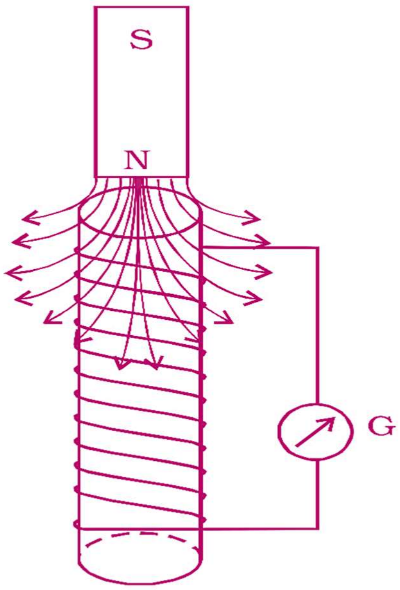
If the coil has N number of turns and φ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time is

$$\boxed{\varepsilon(\text{instantaneous}) = -N\frac{d\phi}{dt}, \qquad \varepsilon_{avg} = -N\frac{(\varphi_2 - \varphi_1)}{t}}$$

Lenz's law

Lenz's law states that the induced current produced in a circuit always flows in such a direction that it opposes the change that produces it.

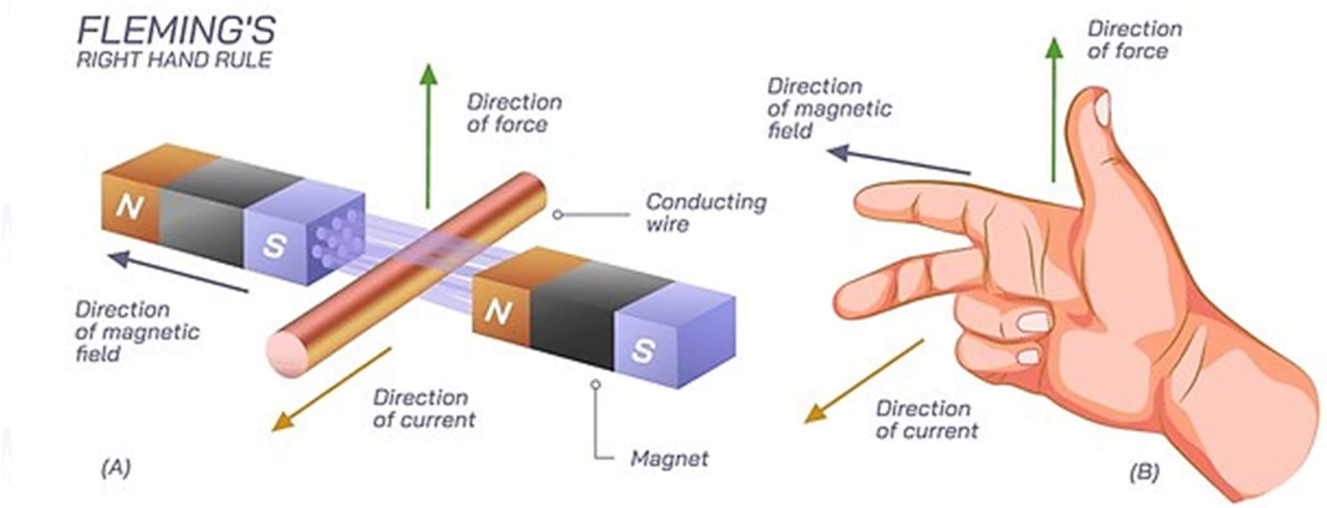
Lenz's law - a consequence of conservation of energy



Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer. A magnet is moved towards the coil. The upper face of the coil acquires north polarity. Consequently, work has to be done to move the magnet further against the force of repulsion. When we withdraw the magnet away from the coil, its upper face acquires south polarity. Now the work done is against the force of attraction. When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current to flow through the coil. The direction of the induced current, according to Lenz's law is always to oppose the motion of the magnet. The work done in moving the magnet is converted into electrical energy.

Fleming's right hand rule

The direction of induced current can be found using this rule. It states that:

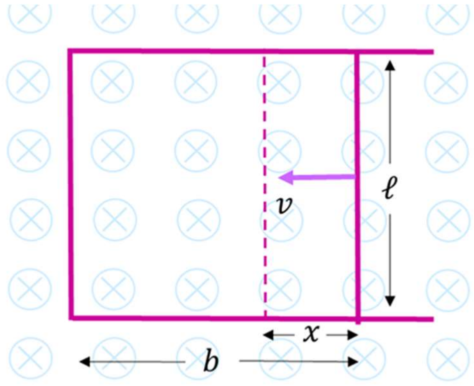


The forefinger, the middle finger and the thumb of the right hand are held in the three mutually perpendicular directions. If the forefinger points along the direction of the magnetic field and the thumb is along the direction of motion of the conductor, then the middle finger points in the direction of the induced current.

Motional emf

In case of a conductor in translational motion in a magnetic field

Figure shows a rectangular conducting loop PQRS in the plane of paper. The conductor is free to move. Let the conductor QR be moved towards right with a constant velocity v. The area enclosed by the loop PQRS increases.



Therefore, amount of magnetic flux linked with the loop increases. An e.m.f. is induced in the loop.
If the length QR = l and the distance through which is it pulled is *x,* then emf induced between ends Q and R is , $\varepsilon = -\frac{d\phi}{dt}$

$$\therefore \varepsilon = -\frac{d}{dt}(BA_2 - BA_1) = -B\frac{d}{dt}(A_2 - A_1) = -B\frac{d}{dt}((b-x)\ell - b\ell)$$

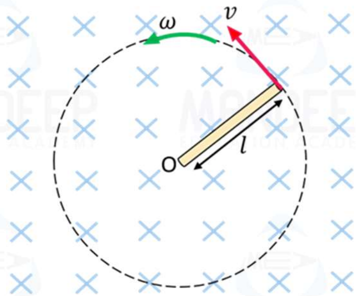
$$\Rightarrow \varepsilon = -B\frac{d}{dt}(b\ell - x\ell - b\ell) = B\ell\frac{dx}{dt} = B\ell v$$

$$\Rightarrow \boxed{\varepsilon = B\ell v}$$

This is called motional emf as it is produced due to motion of a conductor in a magnetic field.

In case of a conductor in rotational motion in a magnetic field

Average linear velocity of the rod is $v_{avg} = \frac{0+\omega\ell}{2} = \frac{\omega\ell}{2}$ therefore, magnitude of emf induced between the ends of the rod is

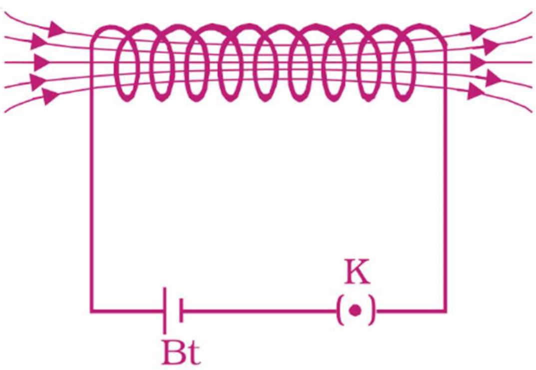


$$\varepsilon = B\ell\left(\frac{\omega\ell}{2}\right)$$

$$\Rightarrow \boxed{\varepsilon = \frac{1}{2}B\ell^2\omega}$$

Self-induction

It is the property of a coil by virtue of which it resists any change in the strength of current flowing through it by inducing an emf in itself.



A coil is connected in series with a battery and a key (K). On pressing the key, the current through the coil increases to a maximum value and correspondingly the magnetic flux linked with the coil also increases. An induced current flows through the coil, which according to Lenz's law opposes the further growth of current in the coil. On releasing the key, the current through the coil decreases to a zero value and the magnetic flux linked with the coil also decreases.
According to Lenz's law, the induced current will oppose the decay of current in the coil.

Coefficient of self-induction

When a current I flows through a coil, the magnetic flux (φ) linked with the coil is proportional to the current.

$$\phi \propto I$$

$$\boxed{\phi = LI}$$

where L is a constant of proportionality and is called coefficient of self induction or self inductance.

$$\text{If } I = 1A,\ \phi = L \times 1 = L,$$

$$\text{then } L = \phi.$$

Therefore, **coefficient of self induction of a coil is numerically equal to the magnetic flux linked with a coil when unit current flows through it. According to laws of electromagnetic induction.**

As

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt},$$

$$\boxed{\varepsilon = -L\frac{dI}{dt}}$$

SI unit of self inductance if henry, denoted by H.

Definition of 1 henry:

if $\frac{dI}{dt} = 1\,A\,s^{-1}$ and $\varepsilon = 1\,V$, then L = 1 H

Therefore, **coefficient of mutual inductance is said to be one henry when an emf of one volt is induced in the coil when rate of change of current through it is 1 A/s.**

Self inductance of a long solenoid

Let us consider a solenoid of N turns with length l and area of cross section A. It carries a current I. If B is the magnetic field at any point inside the solenoid, then

Magnetic flux per turn = B × area of each turn

But, $B = \frac{\mu_0 NI}{\ell}$

Therefore magnetic flux per turn $= \frac{\mu_0 NIA}{\ell}$

Hence, the total magnetic flux (φ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\phi = \frac{\mu_0 NIA}{\ell} \times N$$

i.e. $\phi = \dfrac{\mu_0 N^2 I A}{\ell}$ .... (i)

If L is the coefficient of self induction of the solenoid, then

$\phi = LI$ .... (ii)

From equations (i) and (ii)

$$LI = \frac{\mu_0 N^2 I A}{\ell}$$

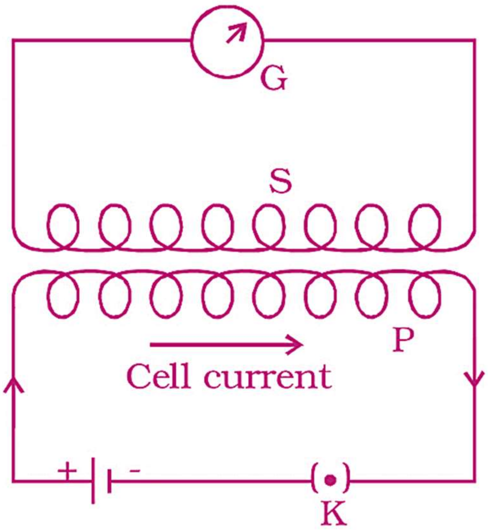
$$\therefore \quad L = \frac{\mu_0 N^2 A}{\ell}$$

If the core is filled with a magnetic material of permeability μ, then

$$L = \frac{\mu N^2 A}{\ell}$$

Mutual induction

It is the property of two coils by which each opposes any change in the strength of current flowing through the other by developing an induced emf.



P and S are two coils placed close to each other. P is connected to a battery through a key K. S is connected to a galvanometer G. On pressing K, current in P starts increasing from zero to a maximum value. As the flow of current increases, the magnetic flux linked with P increases. Therefore, magnetic flux linked with S also increases producing an induced emf in S. Now, the galvanometer shows the deflection. According to Lenz's law the induced current in S would oppose the increase in current in P by flowing in a direction opposite to the current in P, thus delaying the growth of current to the maximum value. When the key 'K' is released, current starts decreasing from maximum to zero value, consequently magnetic flux linked with P decreases. Therefore, magnetic flux linked with S also decreases and hence, an emf is induced in S. According to Lenz's law, the induced current in S flows in such a direction so as to oppose the decrease in current in P thus prolonging the decay of current.

Coefficient of mutual induction

If ϕ_P is the current in coil P and ϕ_S is the magnetic flux linked with coil S due to the current in coil P.

$\therefore \phi_S \propto I_P$ or $\phi_S = MI_P$

where M is a constant of proportionality and is called the coefficient of mutual induction or mutual inductance between the two coils.

If $I_P = 1\,A$, then $M = \phi_S$

Thus, **coefficient of mutual induction of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighbouring coil.**

If ε_S is the induced emf in the coil (S) at any instant of time, then from the laws of electromagnetic induction,

$$MI_1 = \left(\frac{\mu_o N_1 N_2 I_1}{\ell}\right) A$$

$$\therefore M = \frac{\mu_o N_1 N_2 A}{\ell}$$

If the core is filled with a magnetic material of permeability, $M = \frac{\mu N_1 N_2 A}{\ell}$

Thus, **the coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity. The unit of coefficient of mutual induction is henry.**

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

The coefficient of mutual induction between a pair of coils depends on the following factors

1) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.
2) Proximity of the coils

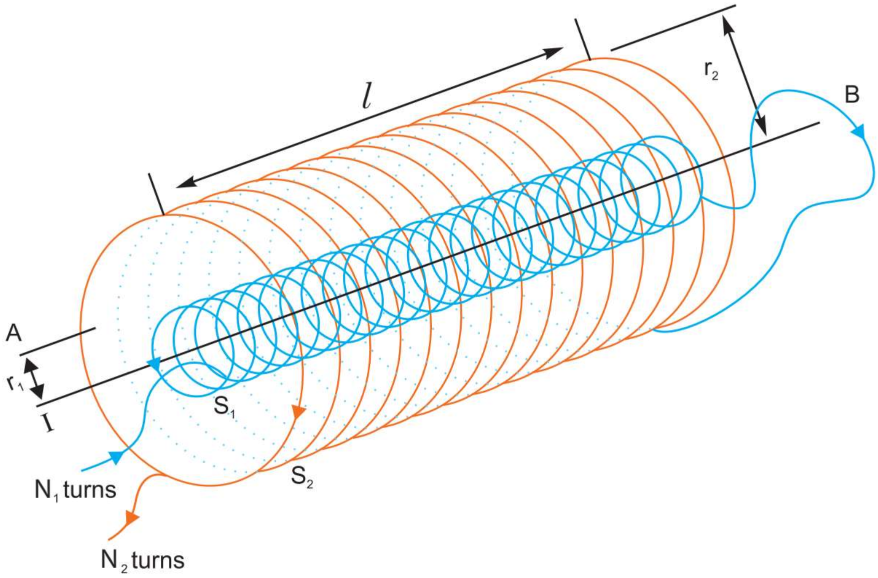
Mutual induction of two long solenoids

S_1 and S_2 are two long solenoids each of length l. The solenoid S_2 is wound closely over the solenoid S_1. N_1 and N_2 are the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1. The magnetic field B_1 produced at any point inside the solenoid S_1 due to the current I_1 is

$$B_1 = \frac{\mu_o N_1 I_1}{\ell} \qquad \text{....... (1)}$$

The magnetic flux linked with each turn of S_2 is equal to B_1A.Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1 A N_2$$



Substituting for B_1 from equation (1)

$$\phi_2 = \left(\frac{\mu_o N_1 I_1}{\ell}\right) A N_2$$

$$\phi_2 = \left(\frac{\mu_o N_1 N_2 I_1}{\ell}\right) A \quad ..... \ (2)$$

$$\text{But } \phi_2 = M I_1 \quad ..... (3)$$

Where M is the coefficient of mutual induction between S_1 and S_2.
From equations (2) and (3)

$$M I_1 = \left(\frac{\mu_o N_1 N_2 I_1}{\ell}\right) A$$

$$\therefore M = \frac{\mu_o N_1 N_2 A}{\ell}$$

If the core is filled with a magnetic material of permeability μ,

$$\boxed{M = \frac{\mu N_1 N_2 A}{\ell}}$$

A is always area of the inner solenoid and ℓ is the length of the longer solenoid if both solenoids have unequal length.

A c generator

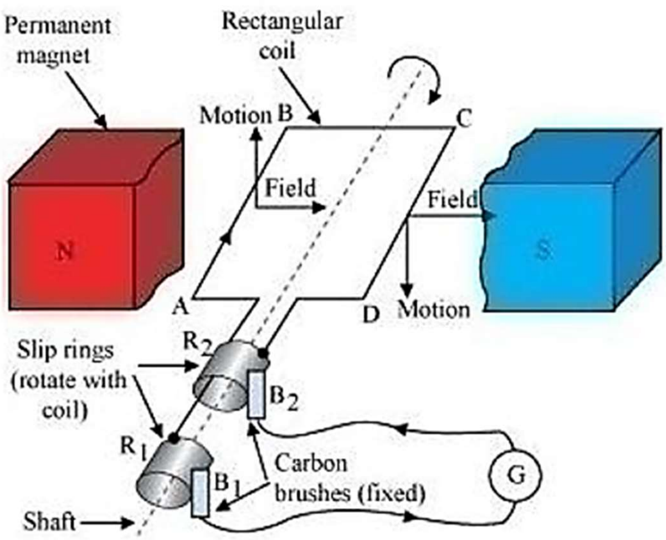
It is a device which is used to convert mechanical energy into electrical energy. Its working is based on the principle of electromagnetic induction.

Construction

It consists of

Armature: It is a coil of large no. of turns of copper wire wound over an iron core. (because iron has high permeability). The coil is rotated about its axis.

Field Magnets: North and south are the pole pieces of a strong magnet in which the coil is rotated. The axis of the coil is perpendicular to the magnetic field.



Rings (R_1 and R_2): (also called slip rings): These are the two hollow metallic rings which are connected to the ends of the coil. These rings rotate with the rotation of coil.

Brushes (B_1 and B_2): These are the two metal plates or carbon rods. They are fixed and kept in contact with R_1 and R_2. The purpose of the brushes is to pass current from the armature to the external load resistance.

Working and theory

Suppose that the plane of the coil is such that its axis is perpendicular to the magnetic field. As the coil is rotated clockwise, the side AB of the coil moves upward and the other side CD moves downwards. The flux in the coil changes, the current is induced, the direction of the current in the side AB will be outward according to Fleming's left-hand rule. In the next half cycle, AB moves downward and CD moves upward. Now, the direction of current induced in AB is inward according to Fleming's left-hand rule. Thus, alternating current is induced in the coil in one complete cycle which flows from the brushes B_1 and B_2 to the external load resistance such that an alternating voltage is developed across R_L.

Let

N be the number of turns in the coil,

B be the magnitude of the induced emf,

A be the area of the coil,

φ be the magnitude of flux linked with the coil, then,

$$E = -\frac{d\varphi}{dt}$$

$$\Rightarrow E = -\frac{d(NBA\cos\omega t)}{dt}$$

$$\Rightarrow E = -NBA\frac{d}{dt}(\cos\omega t)$$

$$\Rightarrow E = -NBA\omega\sin\omega t$$

E is maximum when

$\omega t = 90^\circ$ i.e. when $\sin\omega t = 1$

Let maximum induced emf be E_o

$E_o = NBA\omega$

$\therefore E = E_o \sin\omega t$

Which the magnitude of instantaneous emf induced in the coil. This can be plotted against time as shown.