MOST IMPORTANT QUESTIONS PHYSICS CLASS 11 TERM 2

Some questions are written in red. These questions are not in syllabus of term 2 but if that topic is discussed in your school, then the question is important.

For video lectures of these topics visit "Mandeep Education Academy" YouTube channel

Q1: State and prove Bernoulli's principle or Bernoulli's theorem.

Bernoulli's Principle states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline

Mathematically, it can be expressed as

$$P + \frac{1}{2}\,\text{p}\,\text{v}^2 + \rho\text{gh} = \text{constant}$$

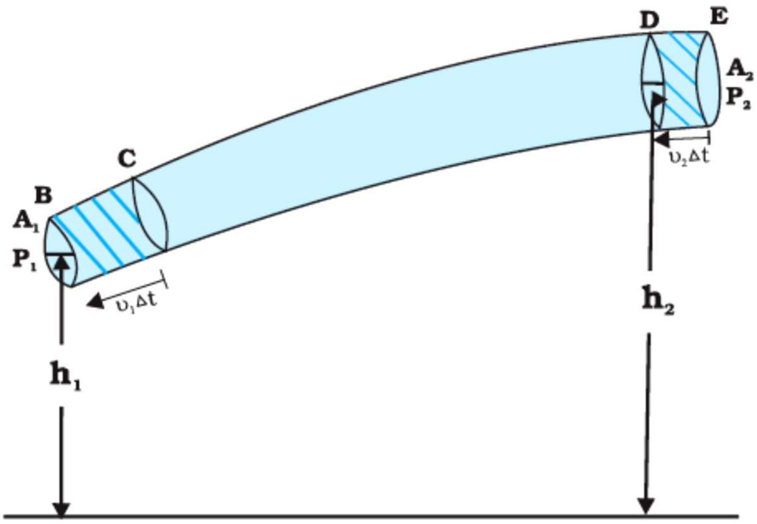
Proof. Consider a non-viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section at A, v_1 the fluid velocity, P_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2 , P_2 and h_2 be the values of the corresponding quantities at B.

Let p be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume x Density

= Area of cross-section x length x densit

Or $m = a_1 v_1 \Delta t\, p = a_2 v_2 \Delta t\, p$



K.E of the fluid

= K.E at B – K.E at A

Or $a_1 v_1 = a_2 v_2$

$\therefore$ Change in

$$= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}a_1 v_1 \Delta t \rho (v_2^2 - v_1^2)$$

Change in P.E of the fluid

= P.E at B – P.E at A

$$= mg(h_2 - h_1) = a_1 v_1 \Delta t \rho g (h_2 - h_1)$$

Net work done on the fluid

= work done on the fluid A – Work done by the fluid at B

= $P_1 a_1 \times v_1 \Delta t - P_2 a_2 \times v_2 \Delta t$

= $P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t$

= $a_1 v_1 \Delta t (P_1 - P_2)$

By conservation of energy,

Net work done on the fluid

= change in K.E of the fluid + change in P.E of the fluid

$\therefore a_1 v_1 \Delta t (P_1 - P_2)$

$$= \frac{1}{2} a_1 v_1 \Delta t \rho (v_2^2 - v_1^2) + a_1 v_1 \Delta t \rho g (h_2 - h_1)$$

Dividing both sides by $a_1 v_1 \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

Or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

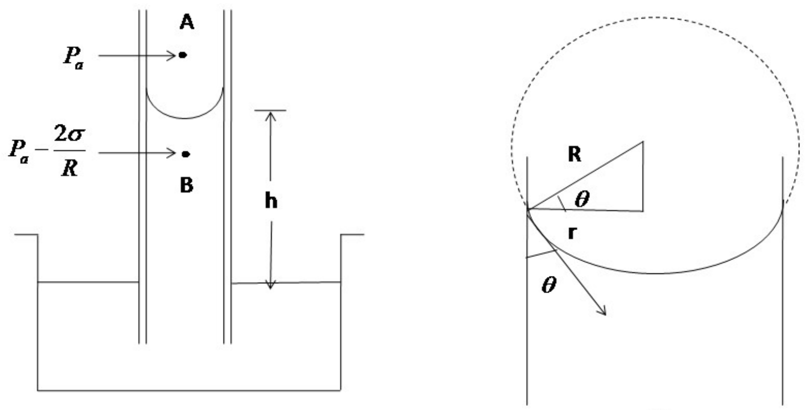
$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Q2: Discuss how a liquid rise or fall in a capillary tube hence derive ascent formula.

Ascent Formula. Consider a capillary tube of radius r dipped in a liquid of surface tension σ and the density p.

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is

$$p = \frac{2\sigma}{R}$$



(a) (b)

Where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in (b), we have

$$\frac{r}{R} = \cos\theta$$

or $$R = \frac{r}{\cos\theta}$$

$\therefore$ $$p = \frac{2\sigma\cos\theta}{r}$$

Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = h\rho g$$

or $$\frac{2\sigma\cos\theta}{r} = h\rho g$$

or $$h = \frac{2\sigma\cos\theta}{r\rho g}$$

This is the **ascent formula** for the rise of liquid in a capillary tube.

Q3: Derive a relation between two principle specific heats of a gas or derive Mayer's formula.

Relation between C_P and C_V: Mayer's formula.

Consider n moles of an ideal gas. Heat the gas to raise their temperature by dT. According to the first law of thermodynamics, the heat supplied dQ is used to partly to increase the internal energy and partly in doing the work of expansion. That is,

$$dQ = dU + PdV$$

If the heat dQ is absorbed at constant volume, then dV = 0 and we have

$$dQ = nC_V dt \text{ and } dQ = dU$$
$$\therefore dU = nC_V dt \qquad \text{................(i)}$$

If now the heat dQ is absorbed at constant pressure, then

$$dQ = dU + PdV$$
$$\Rightarrow nC_P dt = dU + PdV$$

Change in internal energy is same in both case because temperature change is same.

Using (i) we get

$$nC_P dt = nC_V dt + P\Delta V$$
$$\Rightarrow n(C_P - C_V)dt = PdV$$

$$\because PV = nRT$$
$$\therefore PdV = nRdT$$

Putting this in above relation, we get

$$n(C_P - C_V)dt = nRdt$$
$$\text{or} \quad C_P - C_V = R$$

This is the required relation between C_P and C_V. It is also known as Mayer's Formula.

Q4: What is terminal velocity? Derive and expression for the terminal velocity of a body falling freely in a viscous medium.

Terminal Velocity. The maximum constant velocity acquired by a body while falling through a viscous medium is called as Terminal Velocity.

Expression for terminal velocity. Consider a spherical body of radius r falling through a viscous liquid of density of the body.

As the body falls, the various forces acting on the body are:

1. Weight of the body acting vertically downwards.

   $$W = mg = \frac{4}{3}\pi r^3 p\, g$$

2. Upward thrust equal to the weight of the liquid displaced.

   $$U = \frac{4}{3}\pi r^3 \sigma\, g$$

3. Force of viscosity F acting in the upward direction. According to Stoke's Law,

   $$F = 6\pi\eta r v$$

When the body attains terminal velocity v ,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_t = \frac{4}{3}\pi r^3 p\, g$$

Or $$6\pi\eta r v_t = \frac{4}{3}\pi r^3 (p - \sigma) g$$

Or $\qquad v_t = \frac{2}{9} \cdot \frac{r^2(p-\sigma)g}{\eta}$

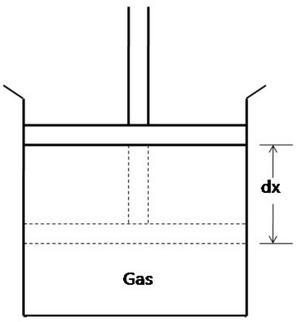
This is the expression for terminal velocity.

Q5: Derive a formula for the work done by an ideal gas in an adiabatic process.

Work done in an adiabatic expansion. Consider n moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let P be the pressure of the gas. When the piston moves up through a small distance dx, the work done by the gas will be

$$dW = PAdx = p\,dV$$

where A is the cross-sectional area of the piston and dV = Adx is the increase in the volume of the gas.



Adiabatic walls

Suppose the gas expands adiabatically and changes from the initial state (P_1, V_1, T_1) to the final state (P_2, V_2, T_2). The total work done by the gas will be

$$W_{adia} = \int_{V_1}^{V_2} PdV$$

For an adiabatic change $PV^{\gamma} = K$ or $P = KV^{-\gamma}$, $\therefore$

$$W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma}dV$$

$$= K\int_{V_1}^{V_2} V^{-\gamma}dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma-1}[KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

But $\quad K = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}]$$

$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$$

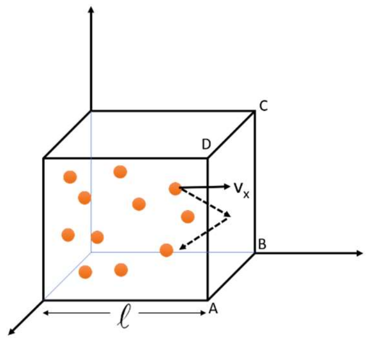
Also, $\quad P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$

$$W_{adia} = \frac{nR}{\gamma-1}[T_1 - T_2]$$

Q6: Derive an expression for the pressure due to an ideal gas

Consider a cubical chamber of edge length ℓ containing an ideal gas as shown. Let number of molecules per unit volume be n. Consider a molecule with velocity v with velocity components v_x, v_y and v_z.

Momentum of this molecule before hitting wall ABCD = mv_x



Since the collisions of an idea gas (according to KTG) are perfectly elastic so the momentum of the molecule after hitting the wall is $-mv_x$ (negative sign because direction is opposite now)

Therefore, change in momentum = $-mv_x - mv_x = -2mv_x$

So, momentum imparted to the wall = $2mv_x$.

Therefore, average momentum of that each molecule imparts to the wall is $2m\bar{v}_x$

where $\bar{v}_x$ is the average of velocity components of molecules in x direction

No of molecules that can hit the wall in time Δt is $n\bar{v}_x\Delta t\ell^2$, but since half of these molecules are moving away from the wall. Therefore, number of molecules that will actually hit the wall in time Δt is $\frac{1}{2}n\bar{v}_x\Delta t\ell^2$.

So, total momentum imparted to wall in time Δt is $\frac{1}{2}n\bar{v}_x\Delta t\ell^2 \times 2m\bar{v}_x = mn\bar{v}_x^2\Delta t\ell^2$

Therefore, force exerted on the wall = $\frac{mn\bar{v}_x^2\Delta t\ell^2}{\Delta t} = mn\bar{v}_x^2\ell^2$

Therefore, pressure exerted by x component, P_x = $\frac{\text{Force}}{\text{Area}} = mn\bar{v}_x^2\ell^2 = \frac{mn\bar{v}_x^2\ell^2}{\ell^2} = mn\bar{v}_x^2$. Since the velocity of gas in all directions should be same due to its random motion, therefore, $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$

Since $\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$ so we get $\bar{v}^2 = 3\bar{v}_x^2 \Rightarrow \bar{v}_x^2 = \frac{1}{3}\bar{v}^2$

Therefore, we get P = $\frac{1}{3}mn\bar{v}^2$. Since mn = ρ (density of gas), therefore

$$\boxed{P = \frac{1}{3}\rho\bar{v}^2}$$

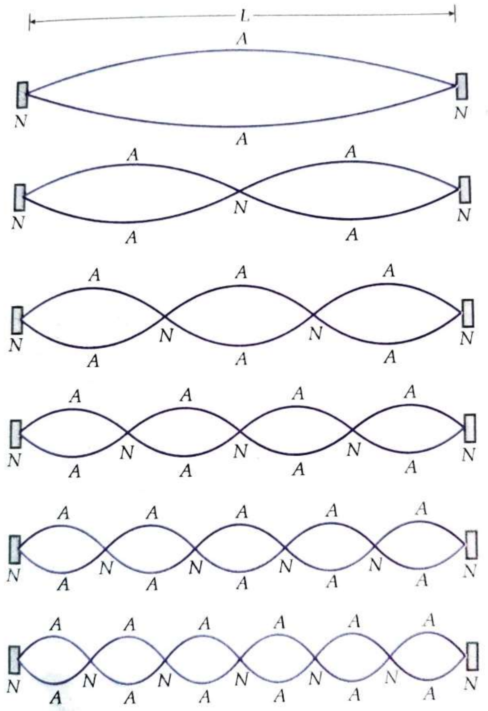
Q7: Discuss the formation of standing waves in a string fixed at both ends and the different modes of vibrations

Or

Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string in the four harmonics are in the ratio 1 : 2: 3 : 4.

Standing waves on stretched strings

Consider a wave travelling along the string given by



$$y_1 = A\sin(\omega t - kx)$$

After reflection from the rigid end the equation of the reflected wave is given by

$$y_2 = A\sin(\omega t + kx + \pi)$$

or

$$y_2 = -A\sin(\omega t + kx)$$

When these two waves superimpose, then the resultant wave is given by

$$y_1 + y_2 = A\sin(\omega t - kx) - A\sin(\omega t + kx)$$

$$y = A\left\{2\sin\left(\frac{\omega t - kx - \omega t - kx}{2}\right)\cos\left(\frac{\omega t - kx + \omega t + kx}{2}\right)\right\}$$

$$y = 2A\sin\left(\frac{-kx}{2}\right)\cos\left(\frac{2\omega t}{2}\right)$$

$$y = -2A\sin kx\cos\omega t$$

As there is always a node at the end, so if length of the rope is L then we can say when x = L, y = 0

$$0 = 2A\sin kL\sin\omega t$$

$$\sin kL = \sin n\pi$$

$$kL = n\pi$$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$L = \frac{n\lambda}{2}$$

For each value of n, there is a corresponding value of λ, so we can write $\frac{2\pi L}{\lambda_n} = n\pi$ or $\lambda = \frac{2L}{n}$

The speed of transverse wave on a string of linear mass density m is given by $v = \sqrt{\frac{T}{m}}$

So the frequency of vibration of the strings is

$$\nu_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For n = 1, $v_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu$ (say)

This is the lowest frequency with which the string can vibrate and is called fundamental frequency or first harmonic.

$$\text{For } n = 2, \quad \nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu \quad \text{(first ovetone or second harmonic)}$$

$$\text{For } n = 3, \quad \nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu \quad \text{(second ovetone or third harmonic)}$$

$$\text{For } n = 2, \quad \nu_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4\nu \quad \text{(third ovetone or fourth harmonic)}$$

Position of nodes

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \ldots\ldots\ldots\ldots, L$$

Position of antinodes

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots\ldots\ldots, \frac{(2n-1)L}{2n}$$

Discuss the formation of standing waves in open and closed organ pipes.

First mode of vibration

In the simplest mode of vibration, there is one node in the middle and to antinodes at the ends of the pipe.

Here length of the pipe,

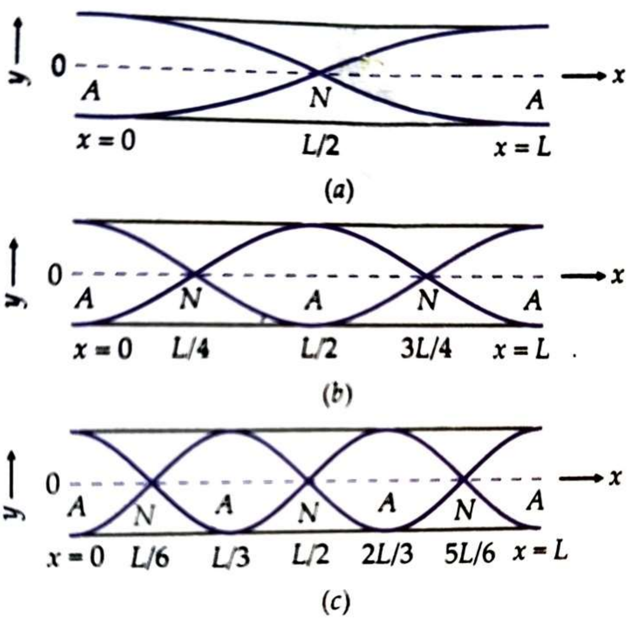
$$L = 2.\frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$

$$\therefore \lambda_1 = 2L$$

Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu$$

This is called fundamental frequency or first harmonic.



Second mode of vibration

Here antinodes at the open ends are separated by two nodes and one antinode.

$$\lambda = 4\frac{\lambda_2}{4} = \lambda_2$$

$$\text{Frequency, } \nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}} = 2\nu$$

This frequency is called first overtone or second harmonic.

Third mode of vibration

Here the antinodes at the open ends are separated by three nodes and two antinodes.

$$L = 6\frac{\lambda_3}{4} \text{ or } \lambda_3 = \frac{2L}{3}$$

$$\therefore \text{Frequency, } \nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called the second harmonic or third harmonic

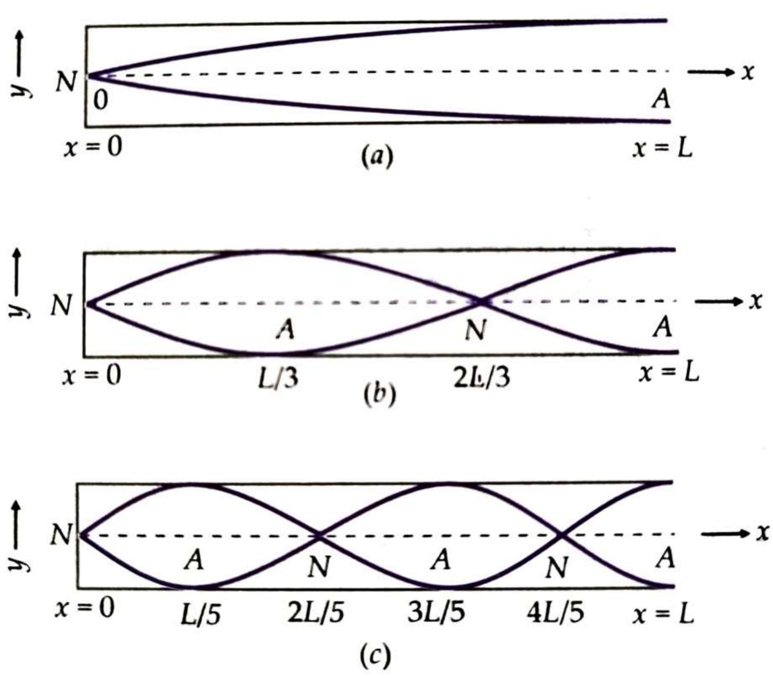
$$\text{Similarly } \nu_n = \frac{v}{\lambda_{3n}} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}} = n\nu$$

Hence the various frequencies of an open organ pipe are in the ratio 1:2:3:4….these are called harmonics.

Closed organ pipes

First mode of vibration

In the simplest mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then



$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L$$

Frequency,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu$$

This is called first harmonic or fundamental frequency.

Second mode of vibration

In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end

$$L = \frac{3\lambda_2}{4} \text{ or } \lambda_2 = \frac{4L}{3}$$

Frequency,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called first overtone or third harmonic.

Third mode of vibration

In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end.

$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

Frequency,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

Hence different frequencies produced in a closed organ pipe are in the ratio 1 : 3 : 5 : 7 …..i.e. only odd harmonics are present in a closed organ pipe.

Q8: Derive an expression for excess pressure inside a liquid drop or soap bubble.

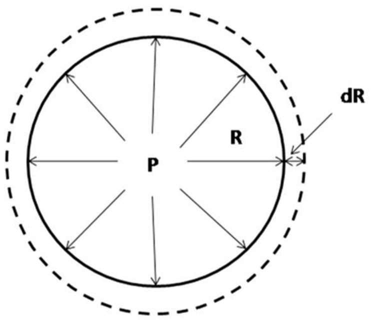
Excess pressure inside a liquid drop. Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to R + dR under the excess pressure p.

Initial surface area = $4\pi R^2$

Final surface area =

$$4\pi(R+dR)^2 = 4\pi(R^2 + 2R\ dR + dR^2)$$
$$= 4\pi R^2 + 8\pi R\ dR$$

dR^2 is neglected as it is small.



Increase in surface area

$= 4\pi R^2 + 8\pi R\ dR - 4\pi R^2 = 8\pi R\ dR$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area x Surface tension

$= 8\pi R\ dR\sigma$

But work done = Force x Distance

= Pressure x Area x Distance

$= p \times 4\pi R^2 \times dR$

Hence, $p \times 4\pi R^2 \times dR = 8\pi R\ dR\sigma$

Excess Pressure,

$$p = \frac{2\sigma}{R}$$

Excess pressure inside a soap bubble. Proceeding as in the case of a liquid drop in the above derivation, we obtain

Increase in surface area = $8\pi R\ dR$

But a soap bubble has air both inside and outside, so it has two free surfaces

$\therefore$ Effective increase in surface area

$= 2 \times 8\pi R\ dR = 16\pi R\ dR$

Work done in enlarging the soap bubble

= Increase in surface energy

= Increase in surface area $\times$ Surface tension

$= 16\pi R\ dR\ \sigma$

But, Work done = Force x Distance

$= p \times 4\pi R^2 \times dR$

Hence

$p \times 4\pi R^2 \times dR = 16\pi R\ dR\ \sigma$

or $$p = \frac{4\sigma}{R}$$

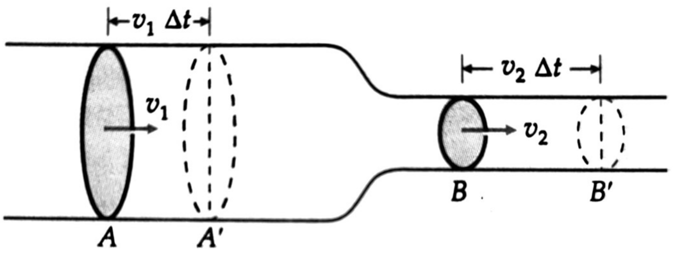
Q9: Derive equation of continuity.

Equation of continuity

Consider a non-viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross section, Let a_1 be the area of cross section, v_1 fluid velocity, ρ_1 fluid density at section A; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.

As m = volume x density

= area of cross section x length x density\



Therefore, mass of fluid that flows through section A in time Δt,

$$m_1 = a_1 v_1 \Delta t \rho_1$$

Mass of fluid that flows through section B in time Δt,

$$m_2 = a_2 v_2 \Delta t \rho_2$$

By conservation of mass

$$m_1 = m_2$$

$$a_1 v_1 \Delta t \rho_1 = a_2 v_2 \Delta t \rho_2$$

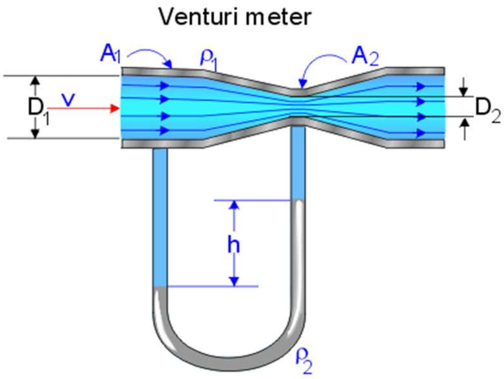
As fluid is incompressible, $\rho_1 = \rho_2$ and hence

$$a_1 v_1 = a_2 v_2$$

Q10: Describe the construction and working of a venturimeter.

Venturimeter. It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle. It is also called flow meter or venturi tube.

Construction. It consists of a horizontal tube having wider opening of cross-section a_1 and a narrow neck of cross-section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density σ.



Working. Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity,

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{v_2}{v_1}$$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) \qquad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right]$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

$$P_1 - P_2 = h\sigma g$$

$$\therefore \quad h\sigma g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

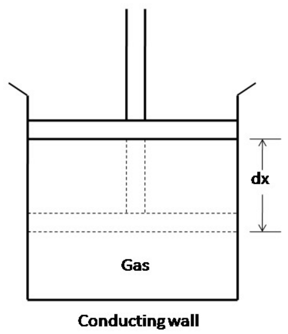
$$\therefore \quad v_1 = \sqrt{\frac{2h\sigma g}{\rho} \times \frac{a_2^2}{a_1^2 - a_2^2}}$$

Volume of the liquid flowing out per second ,

$$Q = a_1 v_1 = a_1 a_2 \sqrt{\frac{2h\sigma g}{\rho(a_1^2 - a_2^2)}}$$

Q11: Derive and expression for work done in an isothermal process by an ideal gas.

Work done in an isothermal expansion. Consider n moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in the figure below. Let P be the pressure of the gas.



Work done by the gas when the piston moves up through a small distance dx is given by

dW = P A dx = PdV

where A is the cross-sectional area of the piston and dV = Adx, is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state (P_1, V_1) to the final state (P_2, V_2). The total amount of work done will be

$$W_{iso} = \int_{V_1}^{V_2} PdV$$

For n moles of a gas, PV = nRT or $P = \frac{nRT}{V}$

$$\therefore W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV = nRT\ [\ln\ V]_{V_1}^{V_2}$$

$$= \text{nRT}\ [\ln \text{V}_2 - \ln\ \text{V}_1] = nRT\ \ln\ \frac{V_2}{V_1}$$

$$or\ W_{iso} = 2.303\ \text{nRT}\ \log\ \frac{V_2}{V_1} = 2.303\ \text{nRT}\ \log\ \frac{P_1}{P_2}$$

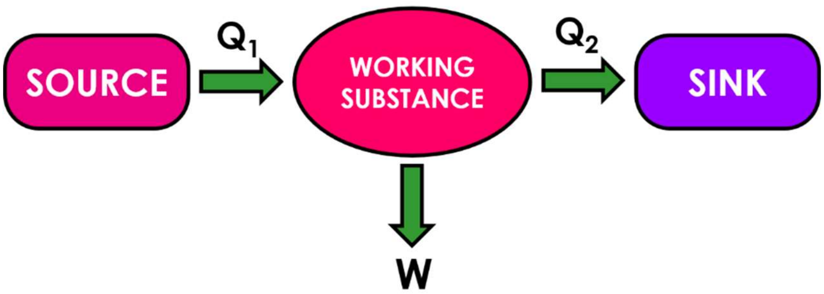
Q12: Describe the construction and working of heat engine and derive an expression for its efficiency.

Heat Engine is a device which converts heat energy into mechanical energy. It has three main parts

1. **Source**: It is a hot reservoir from which a working substance absorbs heat to perform work. It is maintained at a constant temperature T_1.
2. **Working substance:** The substance which absorbs heat from source and performs work is called working substance. For example, a mixture of fuel vapour and air in a gasoline or diesel engine or steam in a steam engine are the working substances.
3. **Sink:** It is the cold reservoir in which the extra heat is rejected after doing work. It is maintained at a constant temperature T_2.

Let heat absorbed from source be Q_1 and heat rejected into the sink be Q_2 after performing W work. Then

$$W = Q_1 - Q_2$$



Efficiency of engine: It the fraction or percentage of energy absorbed which is converted into output mechanical work. It is given by

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Percentage efficiency is given by $\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$

$$\because \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

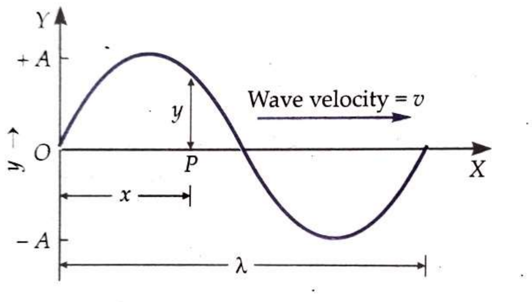
Q13: Derive equation for plane progressive wave.

Suppose a simple harmonic wave starts from the origin O and travels along the positive direction of X-axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at the origin O (x = 0) at any instant t is given by

$y(0,t) = A\sin\omega t$ .... (i)

Where T is the periodic time and A is the amplitude of the wave.

Consider a particle P on x axis at a distance x from O. The disturbance starting from the origin O will reach P in $\frac{x}{v}$ seconds later than the particle at O. Therefore



Displacement of the particle at P at any instant t = Displacement of the particle at O at a $\frac{x}{v}$ seconds earlier

= Displacement of the particle at O at time $\left(t - \frac{x}{v}\right)$

Thus the displacement of the particle at P at any time t can be obtained by replacing t by $\left(t - \frac{x}{v}\right)$ in equation (i)

$$y(x,t) = A\sin\omega\left(t - \frac{x}{v}\right) = A\sin\left(\omega t - \frac{\omega}{v}x\right)$$

But $\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$

The quantity $k = \frac{2\pi}{\lambda}$ is called angular wave number. Hence,

$$\boxed{y(x,t) = A\sin(\omega t - kx)}$$

Q14: Derive Newton's formula for velocity of sound in air and discuss Laplace's correction.

Newton's formula

Newton assumed that the sound waves travel through air under isothermal conditions. He argued that the small amount of heat produced in a compression is rapidly conducted to the surrounding rarefaction where slight cooling is produced. Thus the temperature of the gas remains constant.

For isothermal change

PV = constant

Differentiating both side, we get

$$PdV + VdP = 0$$
$$\Rightarrow PdV = -VdP$$
$$\Rightarrow P = -\frac{dPV}{dV} = B$$

Where B is the bulk modulus of the gas.

Now, since velocity v of a longitudinal wave in medium is given by $v = \sqrt{\frac{B}{\rho}}$, where ρ is the density of the medium, therefore

$$v = \sqrt{\frac{P}{\rho}} = v = \sqrt{\frac{101325}{1.293}} = 280\,ms^{-1}$$

Which is incorrect having an error of 16%

Laplace's correction

Laplace pointed out that the sound travels through a gas under adiabatic conditions not under isothermal conditions because

- Compression and rarefactions are so rapid that there is no time for exchange of heat.
- Air is an insulator so free exchange of heat is not possible.

So, applying the equation of state for an adiabatic process, we get

$$PV^{\gamma} = K$$
$$\Rightarrow P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0$$
$$\Rightarrow \gamma P\frac{V^{\gamma}}{V}dV + V^{\gamma}dP = 0$$
$$\Rightarrow V^{\gamma}\left[\frac{\gamma P}{V}dV + dP\right] = 0$$
$$\Rightarrow \gamma P = -\frac{dPV}{dV} = B$$

$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{1.4 \times 280}\, ms^{-1} = 331.3 ms^{-1}$, which the correct value of velocity of sound in air.

Other important questions

1. What are beats? Derive an expression for beat frequency and beat interval.
2. Derive an expression for pressure at a depth h.
3. State and prove Torricelli's law.
4. What is mean free path? Derive an expression for it.
5. What is Doppler's effect? Derive an expression for apparent frequency heard by listener
6. Derive an expression for displacement, velocity, acceleration, energy and time period of a particle executing SHM.
7. Derive an expression for time period of a simple pendulum.