MOST IMPORTANT QUESTIONS PHYSICS CLASS 11

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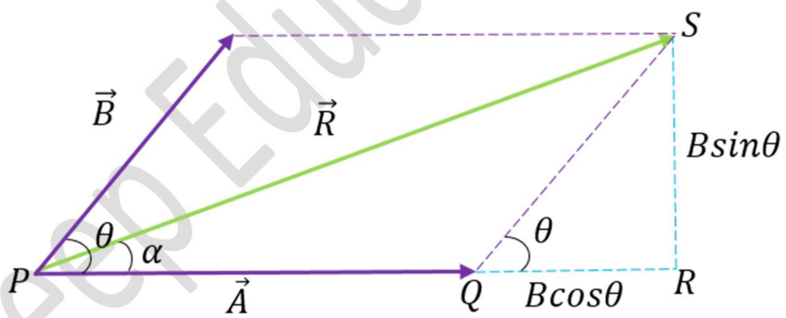
State Parallelogram law of vector addition and derive a formula for magnitude of resultant of two vectors

According to the parallelogram law of vector addition: If two vectors are considered to be the adjacent sides of a parallelogram, then the resultant of the two vectors is given by the vector that is diagonal passing through the point of contact of the two vectors.

Consider two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ as shown. Let their resultant be $\vec{R}$.



Extend PQ and draw QR such that $SR \perp QR$.

In ΔQRS

$$\frac{QR}{QS} = \cos\theta \Rightarrow QR = QS\cos\theta = B\cos\theta \qquad ....(i)$$

$$\frac{SR}{QS} = \sin\theta \Rightarrow SR = QS\sin\theta = B\sin\theta \qquad ....(ii)$$

In ΔPSR

$$(PS)^2 = (PR)^2 + (SR)^2$$

$$\Rightarrow R^2 = (PQ + QR)^2 + (SR)^2$$

$$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2 \qquad \text{[using (i) and (ii)]}$$

$$\Rightarrow R^2 = A^2 + B^2\sin^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then

$$\tan\alpha = \frac{SR}{PR} = \frac{SR}{PQ + QR}$$

$$\Rightarrow \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)}$$

Derive various parameters of a body thrown horizontal direction from a height h.

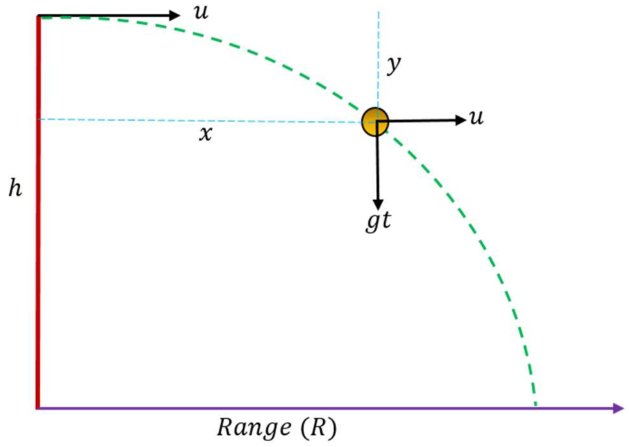
Horizontal projection of projectile

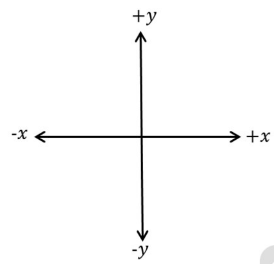
Consider a projectile thrown with velocity u in horizontal direction from a height h as shown

Therefore,

$$u_x = u, \quad u_y = 0$$

$$a_x = 0, \quad a_y = -g$$





Equation of path

$$x = u_x t + \frac{1}{2}a_x t^2 \Rightarrow \boxed{x = ut}$$

$$y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow \boxed{y = -\frac{1}{2}gt^2}$$

$$\because t = \frac{x}{u}$$

$$y = -\frac{1}{2}g\left(\frac{x}{u}\right)^2 = -\frac{1}{2}\frac{g}{u}x^2$$

Which is a quadratic equation. Thus, path of a projectile is parabolic in nature.

Time of flight

Total time for which the projectile remains in air is called time of flight.

$$\because y = u_y t + \frac{1}{2}a_y t^2$$

$$\therefore -h = (0)t - \frac{1}{2}gT^2$$

$$\Rightarrow \boxed{T = \sqrt{\frac{2h}{g}}}$$

Horizontal range (R)

Maximum horizontal distance travelled by projectile.

$$\because x = u_x t + \frac{1}{2} a_x t^2$$

$$\therefore R = uT + \frac{1}{2}(0)T^2$$

$$\Rightarrow \boxed{R = u\sqrt{\frac{2h}{g}}}$$

$$\therefore -h = (0)t - \frac{1}{2}gT$$

Velocity at any instant

$$v_x = u_x + a_x t$$

$$\Rightarrow \boxed{v_x = u}$$

$$v_y = u_y + a_y t$$

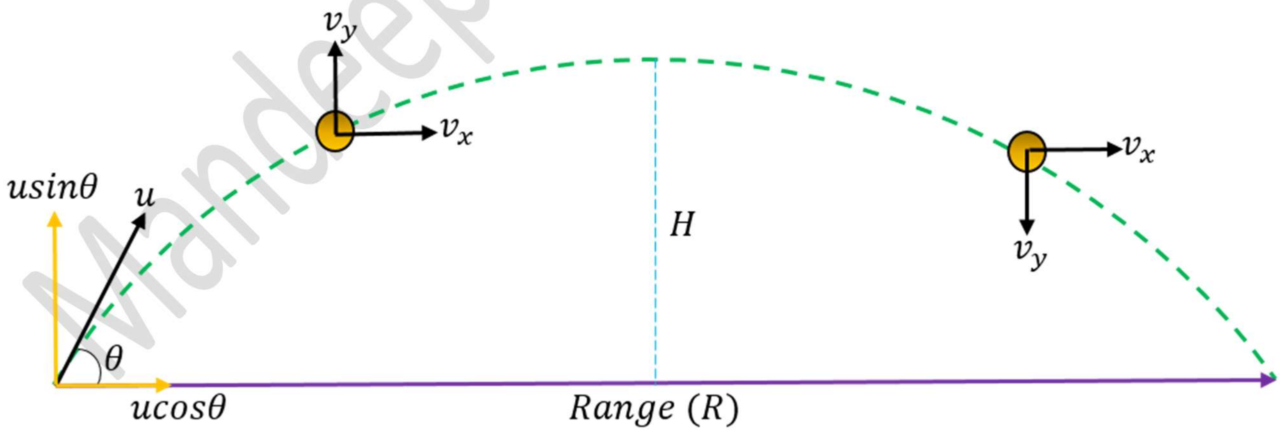
$$\Rightarrow \boxed{v_y = -gt}$$

$$\text{As } v = \sqrt{v_x^2 + v_y^2}$$

$$\boxed{v = \sqrt{u^2 + g^2t^2}}$$

Derive various parameters in angular projectile motion

Consider a body projected with velocity u at an angle θ with horizontal as shown.



Therefore,

$u_x = u\cos\theta,\ u_y = u\sin\theta$

$a_x = 0 \qquad a_y = a\sin\theta$

Equation of path

$$x = u_x t + \frac{1}{2}a_x t^2 \Rightarrow \boxed{x = u\cos\theta t}$$

$$y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow \boxed{y = u\sin\theta t - \frac{1}{2}gt^2}$$

$$\because t = \frac{x}{u\cos\theta}$$

$$\therefore y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = x\tan\theta - \frac{1}{2}\left(\frac{g}{u^2\cos^2\theta}\right)x^2}$$

Time of flight

Total time for which the projectile remains in air is called time of flight.

$$\because y = u_y t + \frac{1}{2}a_y t^2$$

$$\therefore 0 = (u\sin\theta)T - \frac{1}{2}gT^2 \quad [y = 0 \text{ when body hits the ground}]$$

$$\Rightarrow \boxed{T = \frac{2u\sin\theta}{g}}$$

Maximum height attained

At maximum height $v_y = 0$

$$\therefore 0 = u_y + a_y t$$

$$\Rightarrow 0 = u\sin\theta - gt$$

$$\Rightarrow t = \frac{u\sin\theta}{t}$$

Putting this value in equation of y, we get

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow H = u\sin\theta\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{u\sin\theta}{g}\right)^2$$

$$\Rightarrow H = \frac{u^2 \sin^2\theta}{g} - \frac{1}{2}\frac{u^2\sin^2\theta}{g}$$

$$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

Horizontal range

$$\because x = u_x t + \frac{1}{2} a_x t^2$$

$$R = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) + \frac{1}{2}(0)t^2$$

$$\Rightarrow R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$\Rightarrow \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

Velocity at any instant

$$v_x = u_x + a_x t = u\sin\theta$$

$$v_y = u_y + a_y t = u\sin\theta - gt$$

$$\because v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow v = \sqrt{(u\sin\theta)^2 + (u\sin\theta - gt)^2}$$

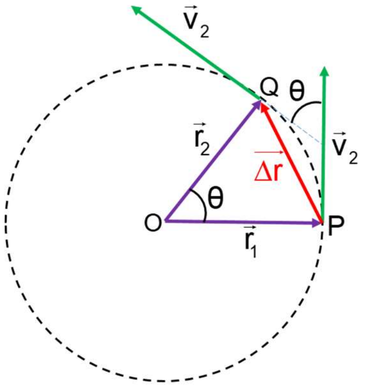
$$\Rightarrow v = \sqrt{u^2\sin^2\theta + u^2\cos^2\theta + g^2t^2 - 2u\sin\theta gt}$$

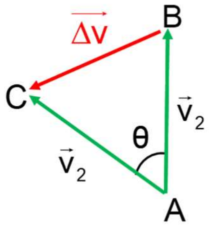
$$\Rightarrow v = \sqrt{u^2\left(\sin^2\theta + \cos^2\theta\right) + g^2t^2 - 2u\sin\theta gt}$$

$$\Rightarrow \boxed{v = \sqrt{u^2 + g^2t^2 - 2u\sin\theta gt}}$$

Derive a formula for centripetal acceleration

Consider a body moving in a circle of radius r with velocity v. Let the position vector of body be $\vec{r}_1$ when it is at P and $\vec{r}_2$ when it is Q. The velocity vector of body at P is $\vec{v}_1$ and Q it is $\vec{v}_2$. If angle between $\vec{r}_1$ and $\vec{r}_2$ is θ then clearly angle between $\vec{v}_1$ and $\vec{v}_2$ is also θ.





Clearly $|\vec{r}_1| = |\vec{r}_2| = r$

Since the motion is uniform so, $|\vec{v}_1| = |\vec{v}_2| = v$

Now in

ΔQOP and ΔCAB

$$\frac{OP}{AB} = \frac{OQ}{AC} = \frac{r}{v}$$

and $\angle QOP = \angle CAB$

$\therefore$ by SAS similarity rule

$\Delta QOP \sim \Delta CAB$

$$\text{so, } \frac{\Delta v}{\Delta r} = \frac{v}{r} \Rightarrow \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right)$$

$$\Rightarrow \boxed{a_c = \frac{v^2}{r}}$$

Discuss the banking of roads and railway tracks and derive a formula for safe turning on a rough banked road.

Outer edge of road and railway tracks are banked so that a component of normal reaction can help the frictional force to provide the necessary centripetal force for the safe turning of vehicles and trains.

Consider a car of mass m moving on a banked road of radius r. The various forces acting on the car are:

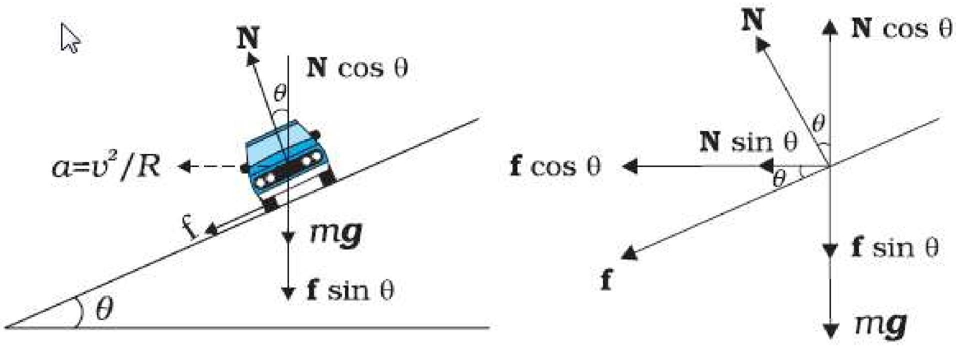
i. Weight (mg) of the car acting in downward direction.
ii. Normal reaction (R) of the road on the car.
iii. For of friction F between the tiers and the road.

Resolve R into two components (i) $N\cos\theta$ and (ii) $N\sin\theta$, similarly $f\cos\theta$ and $f\sin\theta$ are the horizontal and vertical components of the force of friction (f). For the equilibrium of the car

$$mg + f\sin\theta = N\sin\theta$$
$$\Rightarrow mg = N\cos\theta - f\sin\theta$$

$(N\sin\theta + f\cos\theta)$ acts towards the centre of the circular

banked road and provides the n necessary centripetal force to the car



$$N\sin\theta + f\cos\theta = \frac{mv^2}{r}$$

$$\therefore \frac{mv^2}{rmg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \frac{f}{N}\cos\theta}{\cos\theta - \frac{f}{N}\sin\theta}$$

$$\text{Since } \frac{f}{N} = \mu \text{ (coefficient of friction)}$$

$$\therefore \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}}$$

The optimum speed to negotiate a curve can be obtained by putting $\mu = 0$.

$$v = \sqrt{rg\tan\theta}$$

Why does a cyclist bend while taking a circular turn? Explain with the help of necessary calculations.

When a cyclist negotiates a curve, he bends slightly from his vertical position towards the inner side of the curve so that a component of normal reaction can provide the necessary centripetal force. The various forces acting on the system (cycle and man) are:

i. Weight (mg) of the system.
ii. Normal reaction (R) offered by the road to the system and acts at an angle θ with the vertical.

It is assumed that the force of friction between the tyres of the bicycle and the surface is negligible.

Resolve R into two components

Rcosθ which is equal and opposite to the weight (mg) of the system,

Rcosθ = mg ……. (1)

Rsinθ which is directed towards the centre and will provide necessary centripetal force

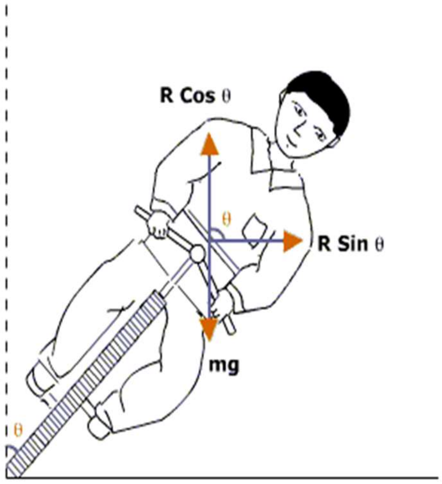
i.e $R\sin\theta = \frac{mv^2}{r}$ ......(2)

Dividing (2) by (1) we get

i.e $\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$

or $\tan\theta = \frac{v^2}{rg}$

$\therefore \boxed{v = \sqrt{rg\tan\theta}}$



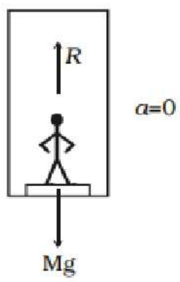
Discuss the concept of apparent weight of a man in an elevator.

let us consider a weighing machine lying on the surface of an elevator or a lift

1. **When the lift is at ret or moving with a constant velocity**

   Forces acting on body are

   a. Weight (mg) of the man acting in downward direction
   b. Normal reaction (R) acting in upward direction



   As the lift is moving with a constant velocity therefore, net force acting on the man is zero hence R = mg, i.e. true weight = apparent weight

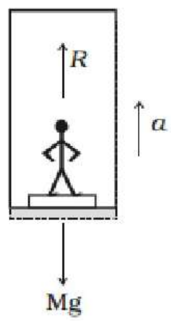
2. **When lift is accelerating**

   In upward direction. If the lift is accelerating in upward direction net force is acting in upward direction i.e. R is more than mg , the equation can be written as

   R - mg = ma

   or R = mg + ma

   i.e. apparent weight > true weight

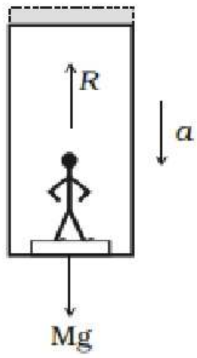


In downward direction. If the lift is accelerating in downward direction net force is acting in downward direction i.e. mg is more than R, the equation can be written as

mg - R = ma

or R = mg - ma

i.e. apparent weight < true weight

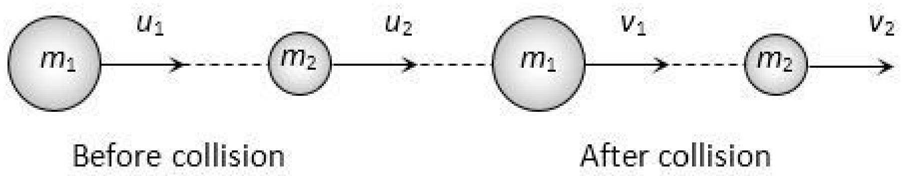


In case of free fall, the acceleration of the lift is g therefore R becomes 0 i.e. apparent weight of the person is zero.

When lift is moving in downward direction with acceleration more than g, then R < 0, i.e. apparent weight of the person becomes negative.

Discuss elastic collision in one dimension. Derive an expression for velocities of two bodies after such a collision.

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 moving in the same straight line colliding with each other. Let their velocities be v_1 and v_2 after the collision.



Since momentum remains conserved in an elastic collision, therefore

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow m_1u_1 - m_1v_1 = +m_2v_2 - m_2u_2$$
$$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ....(i)$$

As kinetic energy is also conserved in elastic collision therefore

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$
$$\Rightarrow \frac{1}{2}m_1u_1^2-\frac{1}{2}m_1v_1^2=\frac{1}{2}m_2v_2^2-\frac{1}{2}m_2u_2^2$$
$$\Rightarrow \frac{1}{2}m_1\left(u_1^2-v_1^2\right)=\frac{1}{2}m_2\left(v_2^2-u_2^2\right)$$
$$\Rightarrow \frac{\cancel{1}}{\cancel{2}}m_1\left(u_1-v_1\right)\left(u_1+v_1\right)=\frac{\cancel{1}}{\cancel{2}}m_2\left(v_2-u_2\right)\left(v_2+u_2\right).......(ii)$$

From (i) and (ii), we get

$$\frac{\cancel{m_1}\,\cancel{(u_1-v_1)}\,(u_1+v_1)}{\cancel{m_1}\,\cancel{(u_1-v_1)}}=\frac{\cancel{m_2}\,\cancel{(v_2-u_2)}\,(v_2+u_2)}{\cancel{m_2}\,\cancel{(v_2-u_2)}}$$
$$\Rightarrow u_1+v_1=v_2+u_2$$
$$\Rightarrow u_1-u_2=v_2-v_1 \quad ....(iii)$$

Thus, relative velocity of approach = relative velocity of separation

Since

$$e\left(\text{coefficient of restitution}\right)=\frac{\text{relative velocity of seperation}}{\text{relative velocity of approach}}$$
$$e=\frac{v_2-v_1}{u_1-u_2}$$

Therefore, for perfectly elastic collision, e = 1

Now, from (iii), we get

$v_2=u_1-u_2+v_1$, putting this in momentum conservation equation, we get

$$m_1u_1+m_2u_2=m_1v_1+m_2\left(u_1-u_2+v_1\right)$$
$$\Rightarrow m_1u_1+m_2u_2=m_1v_1+m_2u_1-m_2u_2+m_2v_1$$
$$\Rightarrow \left(m_1-m_2\right)u_1+2m_2u_2=\left(m_1+m_2\right)v_1$$
$$\Rightarrow \boxed{v_1=\frac{(m_1-m_2)u_1}{(m_1+m_2)}+\frac{2m_2u_2}{(m_1+m_2)}}$$

Similarly, we can prove that

$$\boxed{v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1u_1}{(m_1 + m_2)}}$$

Derive an expression for the elastic potential energy of a stretched spring.

Consider a spring of spring constant k. Let one end of this spring is fixed and a force F is applied on the other end to stretch its length by small amount dx. Then, work done is

$$dW = \vec{F}.\overrightarrow{dx}$$

$$\Rightarrow dW = -(kx)dx\cos 180^\circ$$

$$\Rightarrow dW = -kx(-1)dx$$

$$\Rightarrow dW = kxdx$$

Total work done in stretching the spring from 0 to x_o

$$W = \int_0^{x_o} kxdx$$

$$\Rightarrow W = k\int_0^{x_o} xdx$$

$$\Rightarrow W = k\left[\frac{x^2}{2}\right]_0^{x_o}$$

$$\Rightarrow W = \frac{k}{2}\left[x_o^2 - (0)^2\right]$$

$$\Rightarrow W = \frac{1}{2}kx_o^2$$

This work is stored in the spring in the form of elastic potential energy, so

$$\boxed{U = \frac{1}{2}kx_o^2}$$

State and prove the work energy theorem.

Consider a body of mass m moving with a velocity u. Now let a force F is applied to it and its velocity becomes v after some time. Let the velocity change be dv for small time dt and body travels a distance ds during this time, then work done dW is

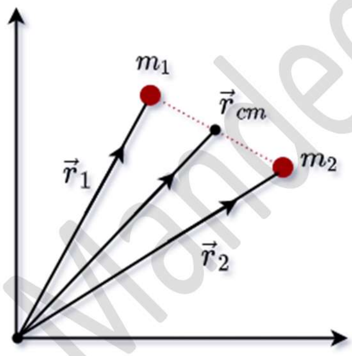
$$dW = Fds$$
$$\Rightarrow dW = (ma)ds$$
$$\Rightarrow dW = m\frac{dv}{dt}ds$$
$$\Rightarrow dW = mdv\left(\frac{ds}{dt}\right)$$
$$\Rightarrow dW = mvdv$$

So, total work done in change velocity from u to v is

$$W = \int_u^v mvdv$$
$$\Rightarrow W = m\left[\frac{v^2}{2}\right]_u^v$$
$$\Rightarrow W = \frac{1}{2}m\left(v^2 - u^2\right)$$
$$\Rightarrow W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$\Rightarrow \boxed{W = KE_{final} - KE_{initial}}$$

Derive a formula for centre of mass of a 2 particle system

Consider a system of two particles P_1 and P_2 of masses m_1 and m_2. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors at any instant t with respect to the origin O, as shown in Fig.



Let m_1, m_2 be masses of two particles
let r_1, r_2 be position vectors of particles
let f_1, f_2 be external forces on particles

Let v_1, v_2 be velocities of particles

Let F_{12}, F_{21} be internal forces on particles (due to each other)

According's to Newton's second law:

$$\vec{f}_1 + \vec{f}_2 + \vec{F}_{12} + \vec{F}_{21} = \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2)$$

As $\vec{v}_1 = \frac{\overrightarrow{dr_1}}{dt}$ and $\vec{v}_2 = \frac{\overrightarrow{dr_2}}{dt}$

$$\vec{f}_1 + \vec{f}_2 + \vec{F}_{12} + \vec{F}_{21} = \frac{d}{dt}\left(m_1\frac{\overrightarrow{dr_1}}{dt} + m_2\frac{\overrightarrow{dr_2}}{dt}\right)$$

But $\vec{F}_{12} = -\vec{F}_{21}$ so they will cancel out

$$\vec{f}_1 + \vec{f}_2 = \frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

Multiply and divide L.H.S by $m_1 + m_2$ we get

$$\vec{f}_1 + \vec{f}_2 = (m_1 + m_2)\frac{d^2}{dt^2}\frac{(m_1\vec{r}_1 + m_2\vec{r}_2)}{(m_1 + m_2)}$$

Let $f = \vec{f}_1 + \vec{f}_2$

$$\vec{f} = (m_1 + m_2)\frac{d^2}{dt^2}\frac{(m_1\vec{r}_1 + m_2\vec{r}_2)}{(m_1 + m_2)}$$

Comparing this equation with $\vec{f} = (m_1 + m_2)\frac{d^2}{dt^2}\vec{R}_{cm}$, we get

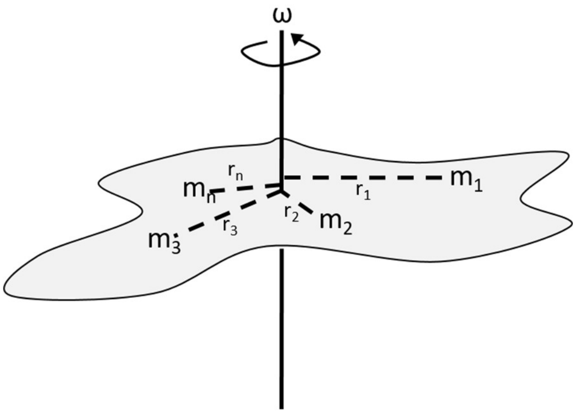
$$\boxed{\vec{R}_{cm} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2)}{(m_1 + m_2)}}$$

Centre of mass of an n particle system

$$\boxed{\vec{R}_{cm} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 \dots\dots\dots + m_n\vec{r}_n)}{(m_1 + m_2 + m_3 + \dots\dots\dots\dots m_n)}}$$

DERIVE A FORMULA FOR MOMENT OF INERTIA

Relation between rotational kinetic energy and moment of inertia. As shown in Fig., consider a rigid body rotating about an axis with uniform angular velocity ω. The body may be assumed to consist of n particles of masses m_1 , m_2 , m_3 ,..........m_n ; situated at distances r_1 , r_2 , r_3 ,..........r_n from the axis of rotation. As the angular velocity ω of all the n particles is same, so their linear velocities are



$v_1 = r_1\,\omega$, $v_2 = r_2\,\omega$, $v_3 = r_3\,\omega$,.........,$v_n = r_n\,\omega$

Hence the total kinetic energy of rotation of the body about the axis of rotation is Rotational K.E.

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots\ldots\ldots + \frac{1}{2}m_nv_n^2$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \ldots\ldots\ldots\ldots\frac{1}{2}m_nr_n^2\omega^2$$

$$= \frac{1}{2}\left(m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots m_nr_n^2\right)\omega^2$$

$$= \frac{1}{2}I\omega^2$$

where $I = m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots m_nr_n^2$ (moment of inertia)

Variation of acceleration due to gravity with height

As acceleration due to gravity at surface of earth, $g = \frac{GM}{R^2}$. Therefore, acceleration due to gravity at a height h above the surface of earth $g' = \frac{GM}{(R+h)^2}$.

$$\therefore \frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} \qquad \text{..........} \quad \text{(i)}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{R^2(1+\frac{h}{R})^2}$$

$$\Rightarrow \frac{g'}{g} = \frac{1}{(1+\frac{h}{R})^2}$$

$$\Rightarrow \frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

If h << R, We can expand above expression using binomial theorem

$$\frac{g'}{g} = \left(1-\frac{2h}{R}\right)$$

$$\Rightarrow 1-\frac{g'}{g} = \frac{2h}{R}$$

$$\Rightarrow \frac{g-g'}{g} \times 100 = \frac{2h}{R} \times 100$$

i.e. percentage decrease in the value of g at a height h = $\frac{2h}{R} \times 100$. Where h is very small as compared to radius of earth.

Variation of g with depth

If body is taken at a depth d below the surface of earth then, $g' = \frac{GM'}{(R-d)^2}$.

Where M' is the mass of that spherical part of earth whose radius is $(R-d)$.

Let earth be a uniform sphere of density ρ, then

$$M=\frac{4}{3}\pi R^3\rho$$

$$\therefore g=\frac{G}{R^2}\frac{4}{3}\pi R^3\rho$$

$$\Rightarrow g=\frac{4}{3}G\pi R\rho$$

And,

$$M'=\frac{4}{3}\pi(R-d)^3\rho$$

$$\therefore g'=\frac{G}{(R-d)^2}\frac{4}{3}\pi(R-d)^3\rho$$

$$\Rightarrow g'=\frac{4}{3}G\pi(R-d)\rho$$

Therefore,

$$\frac{g'}{g}=\frac{\frac{4}{3}G\pi(R-d)\rho}{\frac{4}{3}G\pi R\rho}$$

$$\Rightarrow \frac{g'}{g}=\frac{R-d}{R}\Rightarrow \frac{g'}{g}=1-\frac{d}{R}$$

Hence, percentage decrease at a depth d in the value of g is given by $\left(1-\frac{d}{R}\right)\times 100$.

Derive a formula for escape velocity in terms of parameters of a planet

Consider a body of mass m at a distance x from the centre of earth as shown. Force acting on this body is

$$F=\frac{GMm}{x^2}.$$

Small amount of work (dW) done to move mass m through small distance dx away from earth is $dW = F dx$

$$\Rightarrow dW=\frac{GMm}{x^2}dx$$

Therefore, total work done to move this body from x = R to $x = \infty$ is given by

$$\Rightarrow W = GMm\left[\frac{x^{-2+1}}{-2+1}\right]_R^\infty$$

$$\Rightarrow W = -GMm\left[\frac{1}{x}\right]_R^\infty$$

$$\Rightarrow W = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow W = \frac{GMm}{R}$$

This work must be equal to KE given to the body at the time of launch

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$\Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

Alternate method

Since total energy of the body at the surface and at infinity must be equal. Therefore, we can write

$$\frac{1}{2}mv_e^2 + \left(-\frac{GMm}{R}\right) = 0 + 0$$

$$\Rightarrow \boxed{v_e = \sqrt{\frac{GMm}{R}}}$$

Q1: State and prove Bernoulli's principle or Bernoulli's theorem.

Bernoulli's Principle states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline

Mathematically, it can be expressed as

$$P + \frac{1}{2}\,\text{p}\,v^2 + \rho gh = \text{constant}$$

Proof. Consider a non-viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section at A, v_1 the fluid velocity, P_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2, P_2 and h_2 be the values of the corresponding quantities at B.

Let p be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume x Density

= Area of cross-section x length x density

Or $m = a_1 v_1 \Delta t\, p = a_2 v_2 \Delta t\, p$

K.E of the fluid

= K.E at B – K.E at A

Or $a_1 v_1 = a_2 v_2$

$\therefore$ Change in

$$= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}a_1 v_1 \Delta t \rho (v_2^2 - v_1^2)$$

Change in P.E of the fluid

= P.E at B – P.E at A

$$= mg(h_2 - h_1) = a_1 v_1 \Delta t \rho g (h_2 - h_1)$$

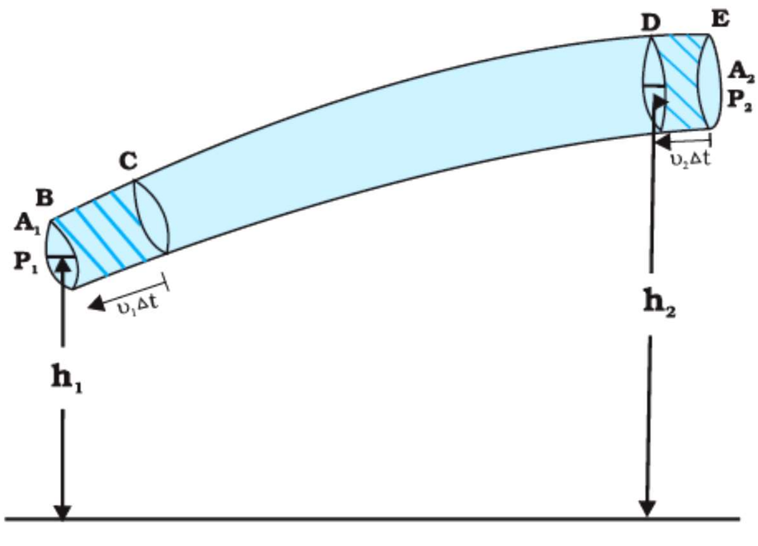
Net work done on the fluid

= work done on the fluid A – Work done by the fluid at B

$$= P_1 a_1 \times v_1 \Delta t - P_2 a_2 \times v_2 \Delta t$$

$$= P_1 a_1 v_1 \Delta t - P_2 a_2 v_2 \Delta t$$

$$= a_1 v_1 \Delta t (P_1 - P_2)$$



By conservation of energy,

Net work done on the fluid

= change in K.E of the fluid + change in P.E of the fluid

$$\therefore a_1 v_1 \Delta t (P_1 - P_2)$$

$$= \frac{1}{2} a_1 v_1 \Delta t \rho (v_2^2 - v_1^2) + a_1 v_1 \Delta t \rho g (h_2 - h_1)$$

Dividing both sides by $a_1 v_1 \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

Or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

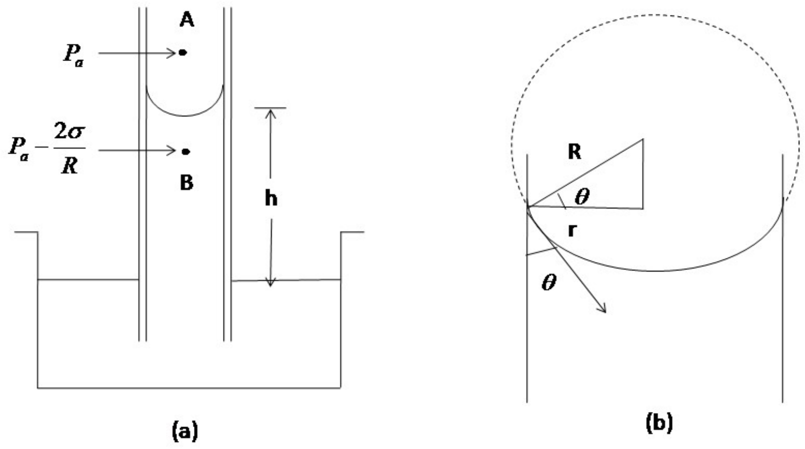
$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Q2: Discuss how a liquid rise or fall in a capillary tube hence derive ascent formula.

Ascent Formula. Consider a capillary tube of radius r dipped in a liquid of surface tension σ and the density p.

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is

$$p = \frac{2\sigma}{R}$$



Where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in (b), we have

$$\frac{r}{R} = \cos\theta$$

or $$R = \frac{r}{\cos\theta}$$

$\therefore$ $$p = \frac{2\sigma\cos\theta}{r}$$

Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = hpg$$

or $$\frac{2\sigma\cos\theta}{r} = hpg$$

or $$h = \frac{2\sigma\cos\theta}{rpg}$$

This is the **ascent formula** for the rise of liquid in a capillary tube.

Q3: Derive a relation between two principle specific heats of a gas or derive Mayer's formula.

Relation between C_P and C_V: Mayer's formula.

Consider n moles of an ideal gas. Heat the gas to raise their temperature by dT. According to the first law of thermodynamics, the heat supplied dQ is used to partly to increase the internal energy and partly in doing the work of expansion. That is,

$$dQ = dU + PdV$$

If the heat dQ is absorbed at constant volume, then dV = 0 and we have

$$dQ = nC_V dt \text{ and } dQ = dU$$
$$\therefore dU = nC_V dt \qquad \text{.................(i)}$$

If now the heat dQ is absorbed at constant pressure, then

$$dQ = dU + PdV$$
$$\Rightarrow nC_P dt = dU + PdV$$

Change in internal energy is same in both case because temperature change is same.

Using (i) we get

$$nC_P dt = nC_V dt + P\Delta V$$
$$\Rightarrow n(C_P - C_v)dt = PdV$$

$$\because PV = nRT$$
$$\therefore PdV = nRdT$$

Putting this in above relation, we get

$$n(C_P - C_v)dt = nRdt$$
$$\text{or} \quad C_P - C_V = R$$

This is the required relation between C_P and C_V. It is also known as Mayer's Formula.

Q4: What is terminal velocity? Derive and expression for the terminal velocity of a body falling freely in a viscous medium.

Terminal Velocity. The maximum constant velocity acquired by a body while falling through a viscous medium is called as Terminal Velocity.

Expression for terminal velocity. Consider a spherical body of radius r falling through a viscous liquid of density of the body.

As the body falls, the various forces acting on the body are:

1. Weight of the body acting vertically downwards.

   $W = mg = \frac{4}{3}\pi r^3 p\, g$

2. Upward thrust equal to the weight of the liquid displaced.

   $U = \frac{4}{3}\pi r^3 \sigma\, g$

3. Force of viscosity F acting in the upward direction. According to Stoke's Law,

   $F = 6\pi\eta r v$

When the body attains terminal velocity v ,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_t = \frac{4}{3}\pi r^3 p\, g$$

Or $$6\pi\eta r v_t = \frac{4}{3}\pi r^3 (p - \sigma) g$$

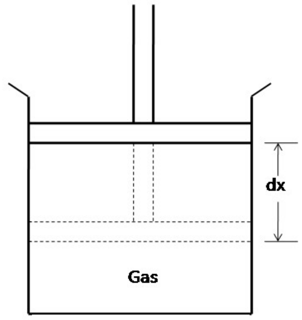
Or $$v_t = \frac{2}{9} \cdot \frac{r^2 (p - \sigma) g}{\eta}$$

This is the expression for terminal velocity.

Q5: Derive a formula for the work done by an ideal gas in an adiabatic process.

Work done in an adiabatic expansion. Consider n moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let P be the pressure of the gas. When the piston moves up through a small distance dx, the work done by the gas will be

$$dW = PAdx = p\,dV$$



Adiabatic walls

where A is the cross-sectional area of the piston and dV = Adx is the increase in the volume of the gas.

Suppose the gas expands adiabatically and changes from the initial state (P_1, V_1, T_1) to the final state (P_2, V_2, T_2). The total work done by the gas will be

$$W_{adia} = \int_{V_1}^{V_2} PdV$$

For an adiabatic change $PV^{\gamma} = K$ or $P = KV^{-\gamma}$, $\therefore$

$$W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma} dV$$

$$= K\int_{V_1}^{V_2} V^{-\gamma} dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma-1}[KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

But $\quad K = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}]$$

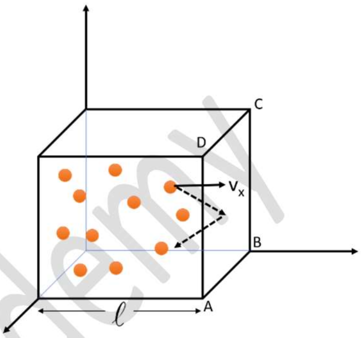
$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$$

Also, $\quad P_1V_1 = nRT_1 \quad$ and $\quad P_2V_2 = nRT_2$

$$W_{adia} = \frac{nR}{\gamma-1}[T_1 - T_2]$$

Q6: Derive an expression for the pressure due to an ideal gas

Consider a cubical chamber of edge length ℓ containing an ideal gas as shown. Let number of molecules per unit volume be n. Consider a molecule with velocity v with velocity components v_x, v_y and v_z.



Momentum of this molecule before hitting wall ABCD = mv_x

Since the collisions of an idea gas (according to KTG) are perfectly elastic so the momentum of the molecule after hitting the wall is $-mv_x$ (negative sign because direction is opposite now)

Therefore, change in momentum = $-mv_x - mv_x$ = $-2mv_x$

So, momentum imparted to the wall = $2mv_x$.

Therefore, average momentum of that each molecule imparts to the wall is $2m\overline{v}_x$

where $\overline{v}_x$ is the average of velocity components of molecules in x direction

No of molecules that can hit the wall in time Δt is $n\overline{v}_x\Delta t\ell^2$, but since half of these molecules are moving away from the wall. Therefore, number of molecules that will actually hit the wall in time Δt is $\frac{1}{2}n\overline{v}_x\Delta t\ell^2$.

So, total momentum imparted to wall in time Δt is $\frac{1}{2}n\overline{v}_x\Delta t\ell^2 \times 2m\overline{v}_x = mn\overline{v}_x^2\Delta t\ell^2$

Therefore, force exerted on the wall = $\frac{mn\overline{v}_x^2\Delta t\ell^2}{\Delta t} = mn\overline{v}_x^2\ell^2$

Therefore, pressure exerted by x component, P_x = $\frac{\text{Force}}{\text{Area}} = mn\overline{v}_x^2\ell^2 = \frac{mn\overline{v}_x^2\ell^2}{\ell^2} = mn\overline{v}_x^2$. Since the velocity of gas in all directions should be same due to its random motion, therefore, $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2$

Since $\overline{v}^2 = \overline{v}_x^2 + \overline{v}_y^2 + \overline{v}_z^2$ so we get $\overline{v}^2 = 3\overline{v}_x^2 \Rightarrow \overline{v}_x^2 = \frac{1}{3}\overline{v}^2$

Therefore, we get P = $\frac{1}{3}mn\overline{v}^2$. Since mn = ρ (density of gas), therefore

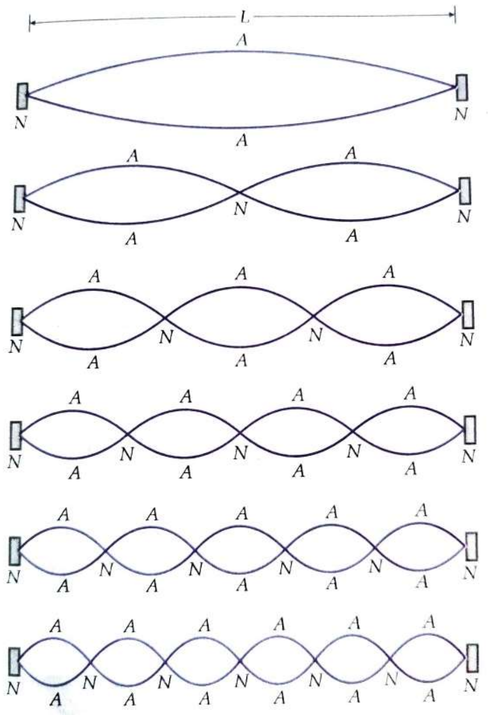
$$\boxed{P = \frac{1}{3}\rho\overline{v}^2}$$

Q7: Discuss the formation of standing waves in a string fixed at both ends and the different modes of vibrations

Or

Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string in the four harmonics are in the ratio 1 : 2: 3 : 4.

Standing waves on stretched strings



Consider a wave travelling along the string given by

$$y_1 = A\sin(\omega t - kx)$$

After reflection from the rigid end the equation of the reflected wave is given by

$$y_2 = A\sin(\omega t + kx + \pi)$$

or

$$y_2 = -A\sin(\omega t + kx)$$

When these two waves superimpose, then the resultant wave is given by

$$y_1 + y_2 = A\sin(\omega t - kx) - A\sin(\omega t + kx)$$

$$y = A\left\{2\sin\left(\frac{\omega t - kx - \omega t - kx}{2}\right)\cos\left(\frac{\omega t - kx + \omega t + kx}{2}\right)\right\}$$

$$y = 2A\sin\left(\frac{-kx}{2}\right)\cos\left(\frac{2\omega t}{2}\right)$$

$$y = -2A\sin kx\cos\omega t$$

As there is always a node at the end, so if length of the rope is L then we can say when x = L, y = 0

$$0 = 2A\sin kL\sin\omega t$$

$$\sin kL = \sin n\pi$$

$$kL = n\pi$$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$L = \frac{n\lambda}{2}$$

For each value of n, there is a corresponding value of λ, so we can write $\frac{2\pi L}{\lambda_n} = n\pi$ or $\lambda = \frac{2L}{n}$

The speed of transverse wave on a string of linear mass density m is given by $v = \sqrt{\frac{T}{m}}$

So the frequency of vibration of the strings is

$$\nu_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For n = 1, $v_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu$ (say)

This is the lowest frequency with which the string can vibrate and is called fundamental frequency or first harmonic.

$$\text{For } n = 2, \quad \nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu \quad \text{(first ovetone or second harmonic)}$$

$$\text{For } n = 3, \quad \nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu \quad \text{(second ovetone or third harmonic)}$$

$$\text{For } n = 2, \quad \nu_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4\nu \quad \text{(third ovetone or fourth harmonic)}$$

Position of nodes

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \ldots\ldots\ldots\ldots, L$$

Position of antinodes

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots\ldots\ldots, \frac{(2n-1)L}{2n}$$

Discuss the formation of standing waves in open and closed organ pipes.

First mode of vibration

In the simplest mode of vibration, there is one node in the middle and to antinodes at the ends of the pipe.

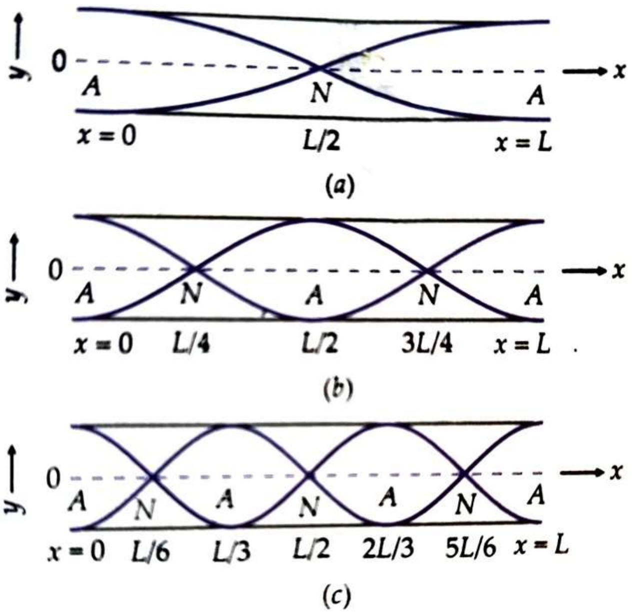
Here length of the pipe,

$$L = 2.\frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$
$$\therefore \lambda_1 = 2L$$

Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu$$

This is called fundamental frequency or first harmonic.



Second mode of vibration

Here antinodes at the open ends are separated by two nodes and one antinode.

$$\lambda = 4\frac{\lambda_2}{4} = \lambda_2$$

$$\text{Frequency, } \nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}} = 2\nu$$

This frequency is called first overtone or second harmonic.

Third mode of vibration

Here the antinodes at the open ends are separated by three nodes and two antinodes.

$$L = 6\frac{\lambda_3}{4} \text{ or } \lambda_3 = \frac{2L}{3}$$

$$\therefore \text{Frequency, } \nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called the second harmonic or third harmonic

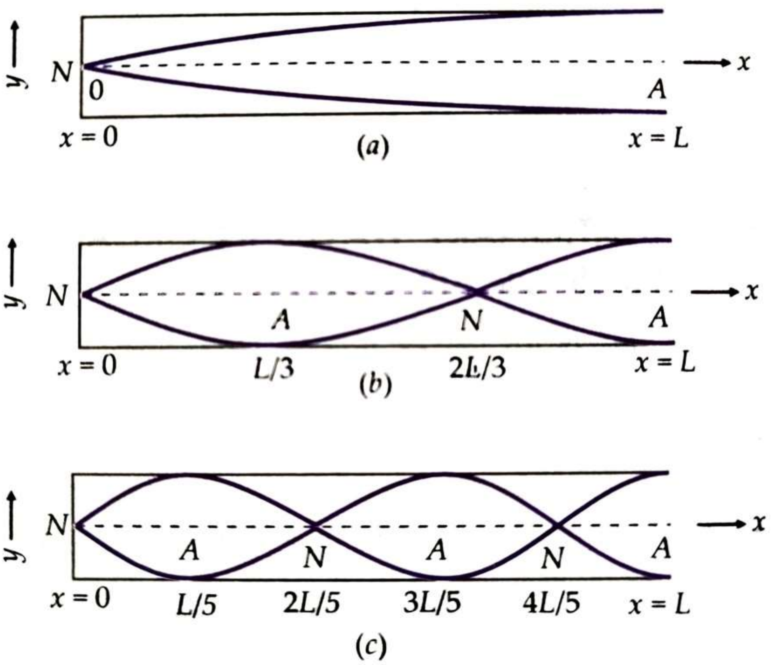
$$\text{Similarly } \nu_n = \frac{v}{\lambda_{3n}} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}} = n\nu$$

Hence the various frequencies of an open organ pipe are in the ratio 1:2:3:4…. these are called harmonics.

Closed organ pipes

First mode of vibration

In the simplest mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then



$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L$$

Frequency,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu$$

This is called first harmonic or fundamental frequency.

Second mode of vibration

In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end

$$L = \frac{3\lambda_2}{4} \text{ or } \lambda_2 = \frac{4L}{3}$$

Frequency,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called first overtone or third harmonic.

Third mode of vibration

In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end.

$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

Frequency,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

Hence different frequencies produced in a closed organ pipe are in the ratio 1 : 3 : 5 : 7 …..i.e. only odd harmonics are present in a closed organ pipe.

Q8: Derive an expression for excess pressure inside a liquid drop or soap bubble.

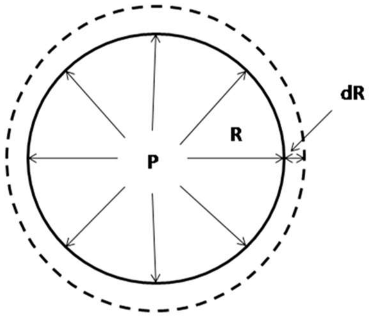
Excess pressure inside a liquid drop. Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to R + dR under the excess pressure p.

Initial surface area = $4\pi R^2$

Final surface area =

$$4\pi(R+dR)^2 = 4\pi(R^2 + 2R\ \mathrm{dR} + \mathrm{dR}^2)$$
$$= 4\pi R^2 + 8\pi R\ dR$$

dR^2 is neglected as it is small.



Increase in surface area

$= 4\pi R^2 + 8\pi R\ dR - 4\pi \mathrm{R}^2 = 8\pi R\ \mathrm{dR}$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area x Surface tension

$= 8\pi R\ \mathrm{dR}\sigma$

But work done = Force x Distance

= Pressure x Area x Distance

$= \mathrm{p} \times 4\pi R^2 \times \mathrm{dR}$

Hence, $\mathrm{p} \times 4\pi R^2 \times \mathrm{dR} = 8\pi R\ \mathrm{dR}\sigma$

Excess Pressure,

$$p = \frac{2\sigma}{R}$$

Excess pressure inside a soap bubble. Proceeding as in the case of a liquid drop in the above derivation, we obtain

Increase in surface area = $8\pi R\ dR$

But a soap bubble has air both inside and outside, so it has two free surfaces

$\therefore$ Effective increase in surface area

$$= 2 \times 8\pi R\, dR = 16\pi R\, dR$$

Work done in enlarging the soap bubble

$$= \text{Increase in surface energy}$$

$$= \text{Increase in surface area} \times \text{Surface tension}$$

$$= 16\pi R\, dR\, \sigma$$

But, Work done = Force x Distance

$$= \text{p} \times 4\pi R^2 \times \text{dR}$$

Hence

$$\text{p} \times 4\pi R^2 \times \text{dR} = 16\pi R\, dR\, \sigma$$

or $$\text{p} = \frac{4\sigma}{R}$$

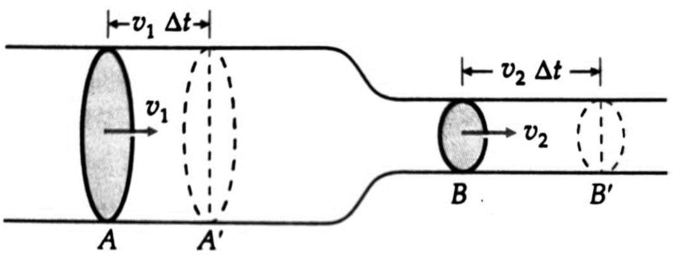
Q9: Derive equation of continuity.

Equation of continuity

Consider a non-viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross section, Let a_1 be the area of cross section, v_1 fluid velocity, ρ_1 fluid density at section A; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.

As m = volume x density

= area of cross section x length x density\



Therefore, mass of fluid that flows through section A in time Δt,

$$m_1 = a_1 v_1 \Delta t \rho_1$$

Mass of fluid that flows through section B in time Δt,

$m_2 = a_2 v_2 \Delta t \rho_2$

By conservation of mass

$m_1 = m_2$

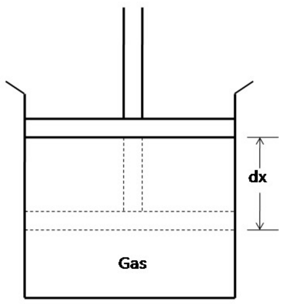
$a_1 v_1 \Delta t \rho_1 = a_2 v_2 \Delta t \rho_2$

As fluid is incompressible, $\rho_1 = \rho_2$ and hence

$a_1 v_1 = a_2 v_2$

Q11: Derive and expression for work done in an isothermal process by an ideal gas.

Work done in an isothermal expansion. Consider n moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in the figure below. Let P be the pressure of the gas.



Work done by the gas when the piston moves up through a small distance dx is given by

dW = P A dx = PdV

where A is the cross-sectional area of the piston and dV = Adx, is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state (P_1, V_1) to the final state (P_2, V_2). The total amount of work done will be

$$W_{iso} = \int_{V_1}^{V_2} P dV$$

For n moles of a gas, PV = nRT or $P = \frac{nRT}{V}$

$$\therefore W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV = nRT \; [\ln \; \mathrm{V}]_{V_1}^{V_2}$$

$$= \mathrm{nRT} \; [\ln \mathrm{V}_2 - \ln \; \mathrm{V}_1] = nRT \; \ln \frac{V_2}{V_1}$$

$$or\ W_{iso} = 2.303\ nRT\ \log \frac{V_2}{V_1} = 2.303\ nRT\ \log \frac{P_1}{P_2}$$

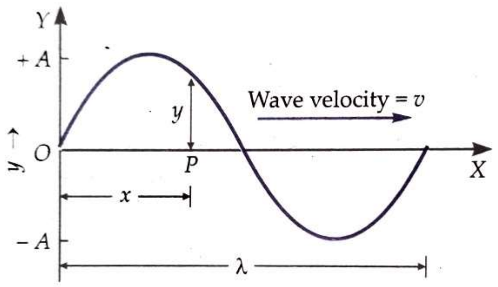
Q13: Derive equation for plane progressive wave.

Suppose a simple harmonic wave starts from the origin O and travels along the positive direction of X-axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at the origin O (x = 0) at any instant t is given by

$y(0,t) = A \sin \omega t$ .... (i)

Where T is the periodic time and A is the amplitude of the wave.

Consider a particle P on x axis at a distance x from O. The disturbance starting from the origin O will reach P in $\frac{x}{v}$ seconds later than the particle at O. Therefore



Displacement of the particle at P at any instant t = Displacement of the particle at O at a $\frac{x}{v}$ seconds earlier

= Displacement of the particle at O at time $\left(t - \frac{x}{v}\right)$

Thus the displacement of the particle at P at any time t can be obtained by replacing t by $\left(t - \frac{x}{v}\right)$ in equation (i)

$$y(x,t) = A\sin\omega\left(t - \frac{x}{v}\right) = A\sin\left(\omega t - \frac{\omega}{v}x\right)$$

But $\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$

The quantity $k = \frac{2\pi}{\lambda}$ is called angular wave number. Hence,

$$\boxed{y(x,t) = A\sin(\omega t - kx)}$$

Other important questions

1. What are beats? Derive an expression for beat frequency and beat interval.
2. Derive an expression for pressure at a depth h.
3. State and prove Torricelli's law.
4. What is mean free path? Derive an expression for it.
5. What is Doppler's effect? Derive an expression for apparent frequency heard by listener
6. Derive an expression for displacement, velocity, acceleration, energy and time period of a particle executing SHM.
7. Derive an expression for time period of a simple pendulum.