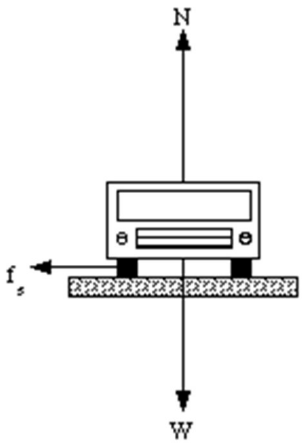
LAWS OF MOTION

What happens when a car turns on an unbanked rough road? What is maximum velocity for same turning on such a road?



Consider a car of mass m is moving with velocity v on a flat horizontal road of radius r. The various forces acting on the road are:

i. Weight (mg) of the car in downward direction.
ii. Normal reaction (N) in the upward direction
iii. Force of friction (f) between the tyres and the road

As vertical forces are balanced therefore, N – mg = 0 or N = mg.

Since the car is moving in a circular path, it requires a centripetal force $F = \frac{mv^2}{r}$.

The centripetal force must be provided by the force of friction between the tyres and the road.

As force of friction

$f = \mu N = \mu mg$

The car remains on road if F = f

If the speed of the car exceeds the speed given by above formula the car will skid and go off the ground (as there won't be enough centripetal force for safe turning).

Thus, for safe turning

$$\frac{\cancel{m}v^2}{r} \geq \mu \cancel{m}g$$

$$v^2 \geq \mu rg$$

$$\Rightarrow \boxed{v \geq \sqrt{\mu rg}}$$

Discuss the banking of roads and railway tracks and derive a formula for safe turning on a rough banked road.

Outer edge of road and railway tracks are banked so that a component of normal reaction can help the frictional force to provide the necessary centripetal force for the safe turning of vehicles and trains.

Consider a car of mass m moving on a banked road of radius r. The various forces acting on the car are:

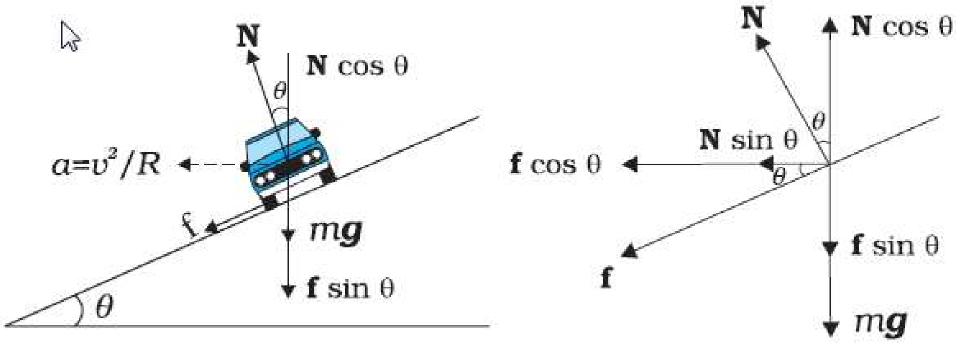
i. Weight (mg) of the car acting in downward direction.
ii. Normal reaction (R) of the road on the car.
iii. For of friction F between the tiers and the road.

Resolve R into two components (i) $N\cos\theta$ and (ii) $N\sin\theta$, similarly $f\cos\theta$ and $f\sin\theta$ are the horizontal and vertical components of the force of friction (f). For the equilibrium of the car

$$mg + f\sin\theta = N\sin\theta$$
$$\Rightarrow mg = N\cos\theta - f\sin\theta$$

$(N\sin\theta + f\cos\theta)$ acts towards the centre of the circular

banked road and provides the n necessary centripetal force to the car



$$N\sin\theta + f\cos\theta = \frac{mv^2}{r}$$
$$\therefore \frac{mv^2}{rmg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \frac{f}{N}\cos\theta}{\cos\theta - \frac{f}{N}\sin\theta}$$

Since $\frac{f}{N} = \mu$ (coefficient of friction)

$$\therefore \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}}$$

The optimum speed to negotiate a curve can be obtained by putting $\mu = 0$.

$$v = \sqrt{rg\tan\theta}$$

Why does a cyclist bend while taking a circular turn? Explain with the help of necessary calculations.

When a cyclist negotiates a curve, he bends slightly from his vertical position towards the inner side of the curve so that a component of normal reaction can provide the necessary centripetal force. The various forces acting on the system (cycle and man) are:

i. Weight (mg) of the system.
ii. Normal reaction (R) offered by the road to the system and acts at an angle θ with the vertical.

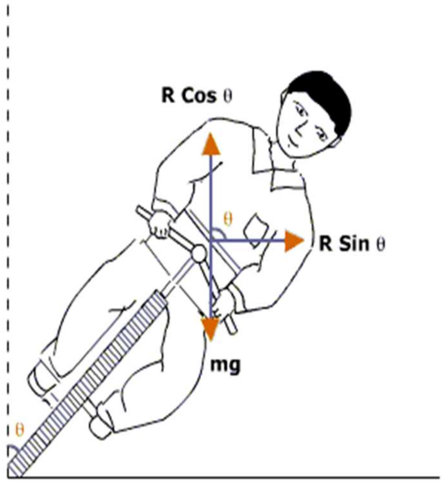
It is assumed that the force of friction between the tyres of the bicycle and the surface is negligible. Resolve R into two components

$R\cos\theta$ which is equal and opposite to the weight (mg) of the system,

$R\cos\theta = mg$ ……. (1)

Rsinθ which is directed towards the centre and will provide necessary centripetal force

i.e $R\sin\theta = \dfrac{mv^2}{r}$ ……(2)



Dividing (2) by (1) we get

$$\text{i.e}\ \frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$$

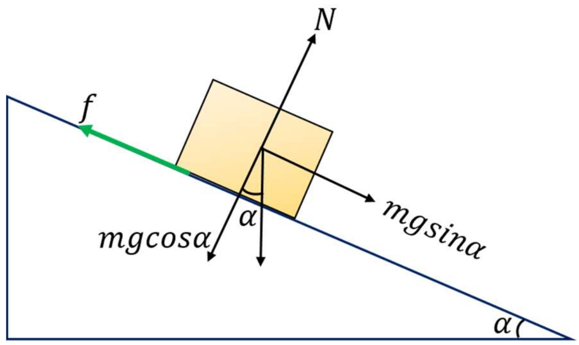
$$\text{or} \quad \tan\theta = \frac{v^2}{rg}$$

$$\therefore \quad \boxed{v = \sqrt{rg\tan\theta}}$$

What is angle of repose? Prove that angle of repose is equal to angle of friction.

The minimum angle made by the inclined plane with the horizontal surface such that the body lying on the inclined plane is just at the verge of sliding down along the inclined plane is called angle of repose.

Let α be the angle made by the inclined plane with the horizontal surface (see fig.). The body will be just in equilibrium, if net force acting on it is zero.



This is possible if f = mgsinα and N = mgcosα

$$\therefore \frac{f}{N} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

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but $\frac{f}{N} = \tan\theta$

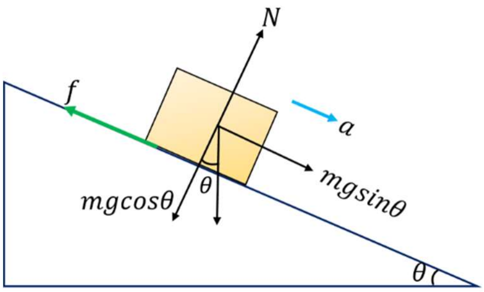
$\therefore \tan\alpha = \tan\theta$

or $\alpha = \theta$

Thus, it is clear from the above discussion that angle of repose = angle of friction.

Derive a formula for the acceleration of a body sliding down a rough inclined plane.

Consider a body of mass m resting on an inclined plane of inclination θ which is greater than angle of repose therefore the body is accelerating downwards. Let a be the acceleration produced in the body. Various forces acting on the body are



i. Weight of the body mg acting vertically downwards.
ii. Normal reaction N which acts vertically upwards.
iii. Force of friction (F) which opposes the relative motion of the body

Now resolve mg into two components

a. mg cosθ which is equal and opposite to the normal reaction to the normal reaction, these two equal and opposite cancel each other

   $\therefore mg\cos\theta = N$ ….(i)

b. mg sinθ which acts downwards along the surface of the inclined plane. This component of the weight acts in a direction opposite to the direction of force of friction. The body accelerates downwards if mg sinθ > F.

   therefore net force acting down the plane is given by

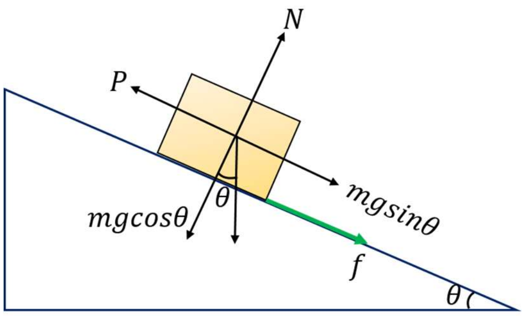
$$mg\sin\theta - f = ma$$
$$\Rightarrow mg\sin\theta - \mu N = ma$$
$$\Rightarrow mg\sin\theta - \mu mg\cos\theta = ma \quad \text{using (i) which is the acceleration of the body}$$
$$\Rightarrow \cancel{m}g(\sin\theta - \mu\cos\theta) = \cancel{m}a$$
$$\Rightarrow \boxed{a = (\sin\theta - \mu\cos\theta)}$$

sliding down the rough inclined plane.

Derive a formula for work done to move a body up a rough inclined plane.

Consider a body of mass m placed over a rough inclined plane having inclination θ with the horizontal. The various forces acting on the body are shown in the figure. As the body is just sliding, therefore, the applied force

$$P = mg\sin\theta + \text{force of friction}$$
$$\Rightarrow P = mg\sin\theta + f$$
$$\Rightarrow f = \mu_k N \text{ or } f = \mu_k mg\cos\theta \quad [\because N = mg\cos\theta]$$
$$\therefore P = mg(\sin\theta + \mu_k\cos\theta)$$

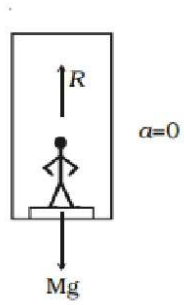


if s is the distance which the body travelled up the plane, then

$$W = P \times s$$
$$\boxed{W = mg(\sin\theta + \mu_k\cos\theta) \times s}$$

Discuss the concept of apparent weight of a man in an elevator.

let us consider a weighing machine lying on the surface of an elevator or a lift



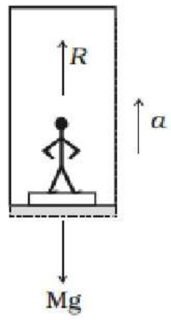
1. **When the lift is at ret or moving with a constant velocity**

   Forces acting on body are

   a. Weight (mg) of the man acting in downward direction
   b. Normal reaction (R) acting in upward direction

   As the lift is moving with a constant velocity therefore, net force acting on the man is zero hence R = mg, i.e. true weight = apparent weight

2. **When lift is accelerating**

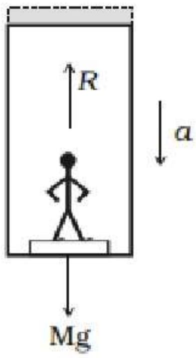


   In upward direction. If the lift is accelerating in upward direction net force is acting in upward direction i.e. R is more than mg , the equation can be written as

   $R - mg = ma$

   or $R = mg + ma$

   i.e. apparent weight > true weight



   In downward direction. If the lift is accelerating in downward direction net force is acting in downward direction i.e. mg is more than R , the equation can be written as

   $mg - R = ma$

   or $R = mg - ma$

   i.e. apparent weight < true weight

   In case of free fall, the acceleration of the lift is g therefore R becomes 0 i.e. apparent weight of the person is zero.

   When lift is moving in downward direction with acceleration more than g, then R < 0, i.e. apparent weight of the person becomes negative.

State and prove the principle of conservation of linear momentum

According to this principle, if net external force acting on a system is zero then total momentum of the system always remains conserved.

Let there are n particles in a system having masses $m_1, m_2, m_3 ....m_n$ respectively and velocities $v_1, v_2, v_3 ....v_n$, then total momentum of the system is

$$P = m_1v_1 + m_2v_2 + m_3v_3.........m_nv_n$$

According to Newton's second law of motion,

$$\text{Force} = \frac{\text{change in momentum}}{\text{time}} \Rightarrow F = \frac{dP}{dt}$$

but if Force acting on the system is zero then,

$$\frac{d}{dt}\left(m_1v_1 + m_2v_2 + m_3v_3.........m_nv_n\right) = 0$$

or $\boxed{m_1v_1 + m_2v_2 + m_3v_3.........m_nv_n = \text{constant}}$

Which is the principle of law of conservation of momentum.

Prove that second law is the real law of motion

Second law is the real law of motion because both first and third law are contained in second law.

First law is contained in second law : According to first law, force is required to produce acceleration in the body and according to second law, $\vec{F} = m\vec{a}$, so if F = 0 then a = 0, which is first law.

Third law is contained in second law.

Consider an isolated system containing two bodies P and Q, let external force acting on the system is 0. Let body P exerts a force F_1 on body Q and Q exerts a force F_2 on body P for the time Δt,

As change in momentum = $\text{force} \times \text{time}$

Therefore change in momentum of P = $F_2 \times \Delta t$ and therefore change in momentum of Q = $F_1 \times \Delta t$

As there no external force acting on the system, therefore according to Newton's second law

Change in momentum =

$$\therefore F_1 \times \Delta t + F_2 \times \Delta t = 0$$
$$\Rightarrow F_1 \times \Delta t = -F_2 \times \Delta t$$
$$\therefore \text{action} = \text{reaction}$$

This is third law.

State newton's second law of motion and derive F = ma.

It states that the rate of change of momentum is directly proportional to the force acting on the body.

Let a body of mass m is moving with a speed u and after a force F is applied let its speed changes to v in time t, then

Initial momentum of the body, P_i = mu and final momentum of the body P_f = mv.
Therefore, change in momentum = P_i - P_f = mv – mu

According to second law

$$\text{Force} = \frac{\text{change in momentum}}{\text{time}} \Rightarrow F = \frac{P_f - P_i}{t}$$

So, according to second law:

$$\therefore F = \frac{mv - mu}{t}$$
$$\Rightarrow F = m\left(\frac{v-u}{t}\right)$$
$$\Rightarrow \boxed{F = ma} \quad \left[\because \frac{(v-u)}{t} = a\right]$$

This is another mathematical statement of second law of motion.

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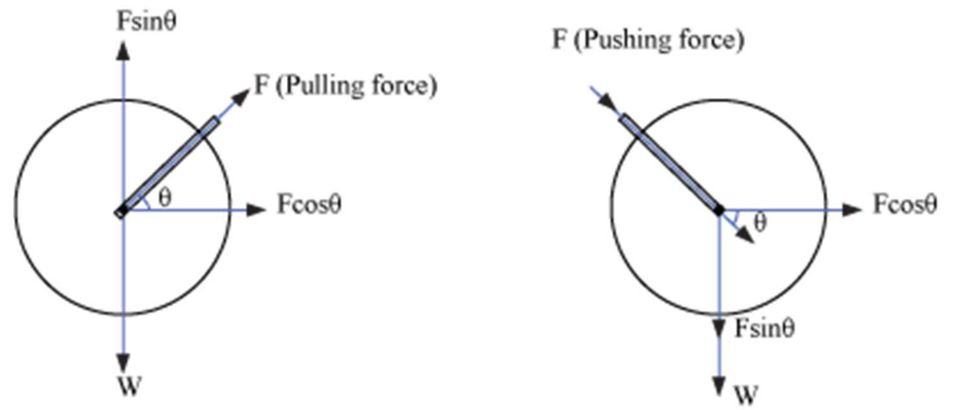
Prove that it is easier to pull a lawn roller than to push it.



Effective weight = W – Fsinθ

Effective weight = W + Fsinθ

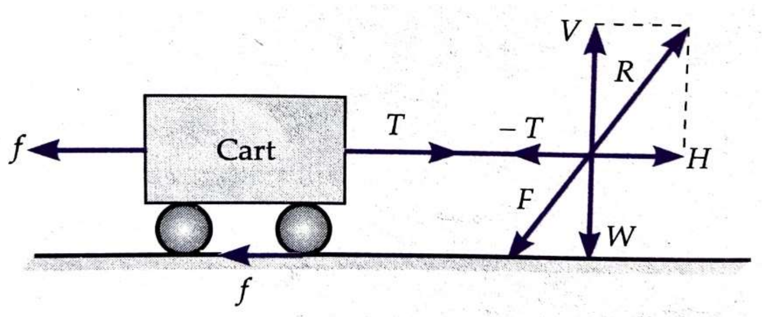
It is clear from the diagram that

During pushing a component of pushing force is in the direction of weight which increases the effective weight $(W + F\sin\theta)$ making the roller feel heavy.

During pulling a component of pulling force is opposite to the direction of weight which decreases the effective weight $(W - F\sin\theta)$ making the roller feel light.

Discuss horse and cart problem

Consider a cart connected to a horse by a string. The horse while pulling the cart produces a tension T in the string in the forward direction (action). The cart, in turn, pulls the horse by an equal force T in the opposite direction.



Initially, the horse presses the ground with a force F in an inclined direction. The direction R of the ground acts on the horse in the opposite direction. The reaction R has two rectangular components:

1. The vertical component V which balances the weight of the horse.
2. The horizontal component H which helps the horse to move forward.

Let f be the force of friction

The horse moves forward in case H > T. In that case net force acting on the horse = H – T

If the acceleration of the horse is a and mass is m, then

$H - T = ma$ .....(i)

The cart moves forward if T > f. In that case,

Net force acting on the cart = $T - f$

The weight of the cart is balanced by the reaction of the ground acting on it.

Since the cart also has same acceleration a. If mass of the cart is M, then

$T - f = Ma$ ....(ii)

Adding (i) and (ii), we get

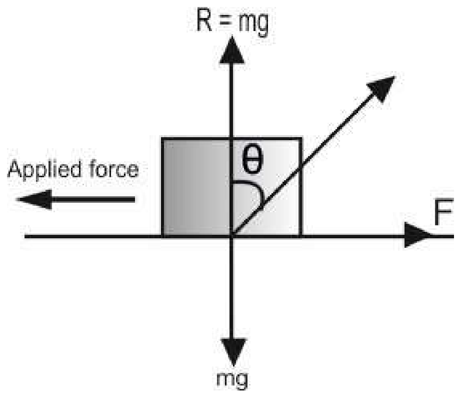
$$H - f = (M + m)a$$
$$\Rightarrow a = \frac{H - f}{M + m}$$

Obviously, a is positive if $H - f$ is positive or if H > f.

Thus, the system moves is H > f i.e. force applied by horse in forward direction is more than the friction between cart and road.

Derive a relation between coefficient of of friction and angle of friction.

The angle between the normal reaction and the resultant of limiting force of friction and normal reaction is called angle of friction.



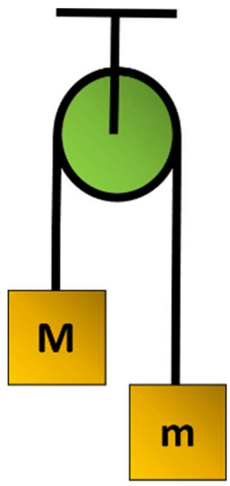
From above figure $\tan\theta = \frac{F}{R}$, but $\frac{F}{R} = \mu_s$

$\therefore \tan\theta = \mu_s$

Thus, co-efficient of static friction is numerically equal to the tangent of the angle of friction.

Derive a formula for acceleration of system and tension in string when two masses are connected on two sides of a pulley.

1. Consider two masses m and M connected to the two free ends of an inextensible string which passes over a smooth pulley. Let T be the tension in the string. The light mass m moves upwards with an acceleration a and the heavy mass M moves downward with an acceleration a.



equation of motion of mass M

Resultant downward force acting on mass M is given by

F = Mg – T ....(i)

But F = ma

therefore Ma = Mg – T

Equation of motion of mass m

Resulting upward for action on mass m is given by

ma = T – Mg ....(ii)

Solving equations (i) and (ii), we get "$a = \frac{(M-m)}{(M+m)}g$,

and putting this value of a in any of the equations we get $T = \frac{2Mg}{M+m}$.

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