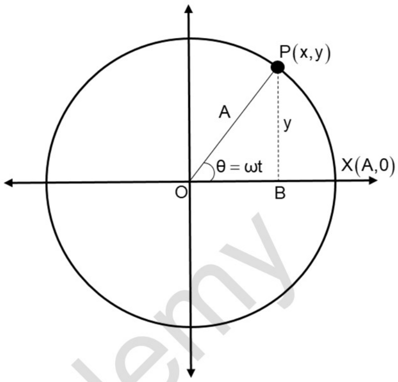
Oscillations all derivations

A body executing SHM can be compared with a body doing circular motion having radius A as shown. Let this body covers an angle θ in time after starting from X (A,0) at t = 0.



In ΔOBP

$$\frac{y}{A} = \sin\theta$$

$$\therefore \boxed{y = A\sin\omega t}$$

This is the displacement equation of a body amplitude of whose motion is A and angular frequency is ω

Velocity in SHM

$$v = \frac{dy}{dt} = A\omega\cos\omega t$$

$$\Rightarrow v = A\omega\sqrt{1-\sin^2\omega t}$$

$$\Rightarrow v = A\omega\sqrt{1-\frac{y^2}{A^2}}$$

$$\Rightarrow v = A\omega\frac{\sqrt{A^2-y^2}}{A}$$

$$\Rightarrow \boxed{v = \omega\sqrt{A^2-y^2}}$$

$$v_{max} = A\omega$$

Acceleration in SHM

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$$

$$\Rightarrow a = -A\omega^2\sin\omega t$$

$$\Rightarrow \boxed{a = -\omega^2 y} \qquad ....(i)$$

Restoring force

$$F = ma$$

$$\Rightarrow \boxed{F = -m\omega^2 y}$$

Time period in SHM

$\because F = -ky$ and also $F = -m\omega^2 y$

$\therefore k = m\omega^2$

So, $\omega = \sqrt{\frac{k}{m}}$

or $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

Also, from i

$\omega = \sqrt{\frac{a}{y}}$

$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{a}{y}}$

$\Rightarrow T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

Kinetic energy in SHM

$KE = \frac{1}{2}mv^2$

$\Rightarrow KE = \frac{1}{2}m\left(\omega\sqrt{A^2 - y^2}\right)^2$

$\Rightarrow \boxed{KE = \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2y^2}$

Potential energy in SHM

$PE = \frac{1}{2}ky^2$

$\Rightarrow \boxed{PE = \frac{1}{2}m\omega^2y^2}$

Total energy in SHM

$TE = PE + KE$

$\Rightarrow TE = \frac{1}{2}m\omega^2y^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2y^2$

$\Rightarrow \boxed{TE = \frac{1}{2}m\omega^2A^2}$

Time period of simple pendulum

Consider a pendulum of length L connected to a bob of mass m as shown. Now from figure it is clear that $mg\sin\theta$ provides the necessary restoring force. Therefore

$$F = -mg\sin\theta$$

for small angles $\sin\theta \approx \theta$

$$\therefore F = -mg\theta$$

$$\theta = \frac{x}{\ell}$$

$$\therefore F = -mg\frac{x}{\ell}$$

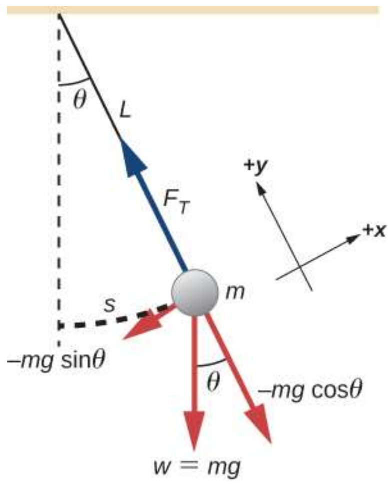
$$\text{or } F = -\left(\frac{mg}{\ell}\right)x$$

$$\therefore k = \frac{mg}{\ell}$$

$$\text{As } T = 2\pi\sqrt{\frac{m}{k}}$$

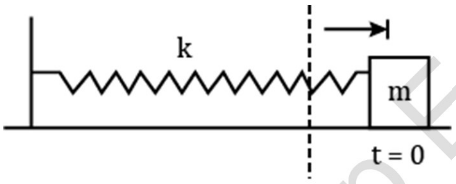
$$\therefore T = 2\pi\sqrt{\frac{m\ell}{mg}}$$

$$\text{or } \boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$



Time period of spring

Horizontal spring



In this spring mass system shown in figure above, time period of oscillation is given by

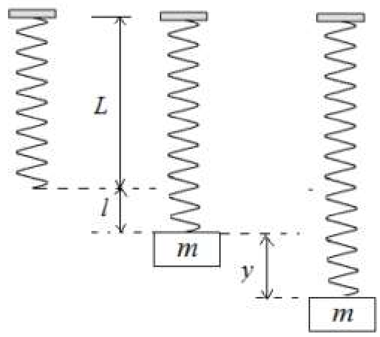
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Vertical spring

When a block is connected to a vertical spring, it extends by an amount ℓ so that the restoring force balances the weight of the block. Therefore,

$$mg = k\ell$$

$$\text{So, } k = \frac{mg}{\ell}$$



$$mg = k\ell$$

$$\text{So, } k = \frac{mg}{\ell}$$

Now, when this spring is pulled by a distance y, it starts doing SHM with time period T which is given by

$$\therefore T = 2\pi\sqrt{\frac{m\ell}{mg}}$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

Combination of springs

Series combination

Consider two springs of spring constants k_1 and k_2 connected in series as shown. Now, when this system oscillates, extensions in springs be y_1 and y_2, then

$$F = -k_1y_1 = -k_2y_2$$

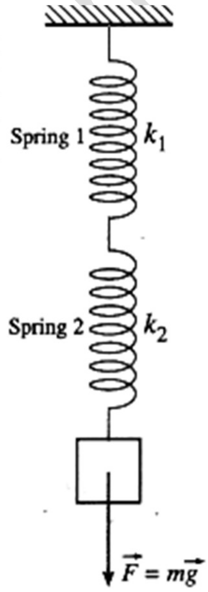
Total extension is

$$y = y_1 + y_2$$

$$\Rightarrow y = -\frac{F}{k_1} - \frac{F}{k_1}$$

$$\Rightarrow y = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow F = -\left(\frac{k_1k_2}{k_1 + k_2}\right)y$$



Comparing it with $F = -kx$, we get

$$k = \frac{k_1k_2}{k_1 + k_2}$$

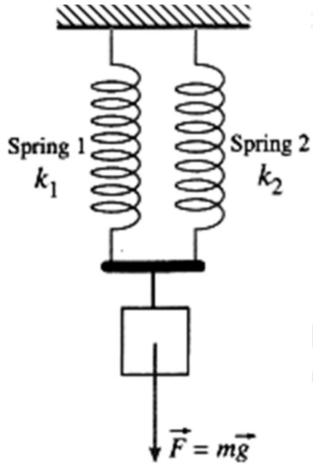
which gives

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}}$$

Parallel combination

Consider two springs of spring constants k_1 and k_2 connected in series as shown. Now, when this system oscillates, extensions in springs be y and restoring forces be F_1 and F_2, then



$$F_1 = -k_1 y,\ F_2 = -k_2 y$$
$$F = F_1 + F_2 = -k_1 y + -k_1 y$$
$$\Rightarrow \boxed{F = -(k_1 + k_2)y}$$
$$\therefore \boxed{k = k_1 + k_2}$$
$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}}$$

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