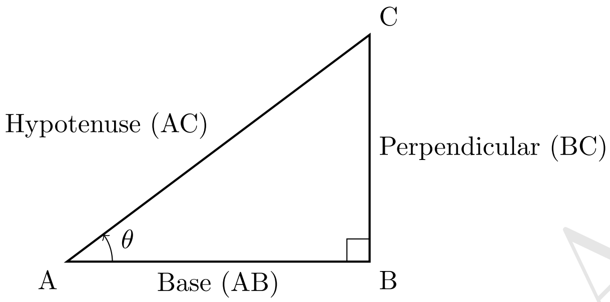
1 Various Trigonometric Ratios

Trigonometry deals with the relationships between the angles and sides of a right-angled triangle. The primary trigonometric ratios are:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}, \quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}, \quad \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}, \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}, \quad \cot\theta = \frac{\text{Base}}{\text{Perpendicular}}$$



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2 Trigonometric Ratios for Standard Angles

The values of trigonometric ratios at standard angles are given in the table below:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

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3 Trigonometric Identities

The three fundamental trigonometric identities are:

3.1 Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

3.2 Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta}, \quad \cos\theta = \frac{1}{\sec\theta}, \quad \tan\theta = \frac{1}{\cot\theta}$$

3.3 Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

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Example Problems

1. Find the value of $\sin 30^\circ + \cos 60^\circ$

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

$$\sin 30^\circ + \cos 60^\circ = \frac{1}{2} + \frac{1}{2} = 1$$

2. Prove the identity:

$$1 + \tan^2\theta = \sec^2\theta$$

Solution:

$$\text{LHS} = 1 + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$= \text{RHS}$$