1 General Form of Linear Equations in Two Variables

A linear equation in two variables is an equation of the form:

$$ax + by + c = 0$$

where:

- a, b are coefficients of variables x and y,
- c is a constant term,
- x, y are variables.

Example: The equation $2x + 3y - 5 = 0$ is a linear equation in two variables.

2 Number of Solutions of a Linear Equation

A linear equation in two variables has infinitely many solutions, as its graphical representation is a straight line. Any point on the line represents a solution.

Example: For the equation $x + y = 5$, solutions include:

$$(0, 5), (1, 4), (2, 3), (3, 2), (4, 1)$$

3 Graph of a Linear Equation

The graph of a linear equation in two variables is always a straight line. Steps to plot the graph:

1. Rewrite the equation in slope-intercept form $y = mx + c$.
2. Choose values for x, compute y.
3. Plot points and draw the line.

Example: For $x + 2y = 6$, plot points:

$$(0, 3), (2, 2), (4, 1)$$

4 Methods to Find Simultaneous Solutions of Linear Equations

To solve two linear equations in two variables, the following methods can be used:

4.1 Graphical Method

- Plot both equations on a graph.
- The intersection point represents the unique solution.

Example: Solve:

$$x + y = 5, \quad 2x - y = 1$$

Solution: Plot both lines; the intersection at $x = 2, y = 3$ is the solution.

4.2 Substitution Method

- Solve one equation for one variable in terms of the other.
- Substitute into the second equation.

Example:

$$x + y = 10, \quad 2x - y = 4$$

Solve $x = 10 - y$, substitute in the second equation:

$$2(10 - y) - y = 4 \Rightarrow y = 8, \quad x = 2$$

4.3 Elimination Method

- Multiply equations to make coefficients equal.
- Subtract or add equations to eliminate one variable.

Example:

$$2x + 3y = 12, \quad 4x - 3y = 6$$

Adding both equations:

$$6x = 18 \Rightarrow x = 3, \quad y = 2$$

4.4 Cross Multiplication Method

For equations:

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Example:

$$3x + 4y = 10, \quad 5x - 2y = 6$$

$$\frac{x}{(4)(6) - (-2)(10)} = \frac{y}{(10)(5) - (6)(3)} = \frac{1}{(3)(-2) - (5)(4)}$$

Solving, we get $x = 2, y = 1$.

5 Consistent and Inconsistent Equations

Two equations can be:

- **Consistent:** If they have at least one solution.
- **Inconsistent:** If they have no solution (parallel lines).

Example: The equations $x + y = 2$ and $2x + 2y = 5$ are inconsistent.

6 Checking Unique, Infinite, and No Solution Using Equations

For equations in the form:

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

- Unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Such equations represent intersecting lines in graph.
- Infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. Such equations represent coincident lines in graph.
- No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Such equations represent parallel lines in graph.

Example:

$$2x + 3y = 5, \quad 4x + 6y = 10$$

Since $\frac{2}{4} = \frac{3}{6} = \frac{5}{10}$, the system has infinitely many solutions.