# 1 Fundamentals of Probability

**Experiment:** An operation that can produce some well-defined outcomes.

**Sample Space:** The set of all possible outcomes of a random experiment.

**Event:** Any subset of the sample space is called an event.

**Probability:** The probability $P(E)$ of an event $E$ is defined as:

$$P(E) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

**Elementary Event:** Each outcome of a random experiment.

**Sure Event (Certain Event):** An event that always occurs whenever the random experiment is performed.

**Impossible Event:** An event that never occurs whenever the random experiment is performed.

**Favourable Event:** The outcomes that ensure the occurrence of an event.

**Complement of an Event:** The complement of event $E$, denoted by $P(\text{not } E)$, occurs only when $E$ does not happen:

$$P(\text{not } E) = 1 - P(E)$$

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# 2 Important Properties of Probability

1. The sum of the probabilities of all elementary events of an experiment is 1:

$$P(E_1) + P(E_2) + P(E_3) + \cdots + P(E_n) = 1$$

2. The probability of a sure event is always 1:

$$P(\text{Sure Event}) = 1$$

3. The probability of an impossible event is always 0:

$$P(\text{Impossible Event}) = 0$$

4. The probability of any event lies between 0 and 1:

$$0 \leq P(E) \leq 1$$

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# 3 Playing Cards and Probability

A deck of 52 cards is divided into 4 suits with 13 cards each. The suits are:

- **Spades (black)**

- **Hearts (red)**

- **Clubs (black)**

- **Diamonds (red)**

**Important Observations:**

- Out of 52 cards, 26 are red and 26 are black.

- Each suit has 13 cards: Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, and 2.

- King, Queen, and Jack are called **Face Cards**, making a total of 12 face cards in a deck.

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# 4 Example Problems

**Example 1:** A fair coin is tossed once. Find the probability of getting a head.
**Solution:**

$$\text{Sample Space} = \{H, T\}$$

$$\text{Favourable Outcomes} = 1 \quad (\text{Getting a Head})$$

$$P(\text{Head}) = \frac{1}{2}$$

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**Example 2:** A die is rolled once. Find the probability of getting an even number.
**Solution:**

$$\text{Sample Space} = \{1, 2, 3, 4, 5, 6\}$$

$$\text{Favourable Outcomes} = \{2, 4, 6\}$$

$$P(\text{Even number}) = \frac{3}{6} = \frac{1}{2}$$

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**Example 3:** Find the probability of drawing a face card from a well-shuffled deck of 52 cards.
**Solution:**

$$\text{Total number of cards} = 52$$

$$\text{Favourable Outcomes} = 12 \quad (4 \text{ suits } \times 3 \text{ face cards each})$$

$$P(\text{Face Card}) = \frac{12}{52} = \frac{3}{13}$$

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# 5 Conclusion

- Probability is the measure of uncertainty of events in random experiments.
- The probability of any event lies between 0 and 1.
- The complement rule helps to calculate the probability of the non-occurrence of an event.
- Problems related to playing cards and dice are commonly used in probability calculations.