1 What are Quadratic Equations?

A quadratic equation is a polynomial equation of degree 2, which can be written in the standard form:

$$ax^2 + bx + c = 0$$

where:

- a, b, c are real numbers.
- $a \neq 0$.

Examples:

$$2x^2 + 3x - 5 = 0, \quad x^2 - 4x + 4 = 0, \quad 3x^2 - 7 = 0$$

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2 Number of Roots of a Quadratic Equation

A quadratic equation always has **two roots** (solutions), which can be:

- Real and distinct
- Real and equal
- Complex (imaginary)

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3 Solving Quadratic Equations by Factorization

Steps:

1. Write the quadratic equation in standard form: $ax^2 + bx + c = 0$.
2. Find two numbers whose product is ac and sum is b.
3. Split the middle term using these numbers.
4. Factorize and solve for x.

Example 1: Solve $x^2 + 5x + 6 = 0$.

$$x^2 + 5x + 6 = 0$$

$$x^2 + 2x + 3x + 6 = 0$$

$$x(x + 2) + 3(x + 2) = 0$$

$$(x + 2)(x + 3) = 0$$

$$\Rightarrow x = -2, \quad x = -3$$

Example 2: Solve $3x^2 - 2x - 8 = 0$.

$$3x^2 - 6x + 4x - 8 = 0$$

$$3x(x - 2) + 4(x - 2) = 0$$

$$(x - 2)(3x + 4) = 0$$

$$\Rightarrow x = 2, \quad x = -\frac{4}{3}$$

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4 Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$ can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve $2x^2 - 4x - 6 = 0$.

$$a = 2, \quad b = -4, \quad c = -6$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{4 \pm \sqrt{64}}{4}$$

$$x = \frac{4 \pm 8}{4}$$

$$x = 3, \quad x = -1$$

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5 Nature of Roots

The nature of the roots of a quadratic equation depends on the value of the discriminant:

$$D = b^2 - 4ac$$

- **If $D > 0$:** Two real and distinct roots.

- **If $D = 0$:** Two real and equal roots.

- **If $D < 0$:** No real roots (complex roots exist).

Example 1: Determine the nature of roots for $x^2 - 6x + 9 = 0$.

$$D = (-6)^2 - 4(1)(9) = 36 - 36 = 0$$

Since $D = 0$, the roots are **real and equal**.
Example 2: Determine the nature of roots for $3x^2 + 4x + 5 = 0$.

$$D = (4)^2 - 4(3)(5) = 16 - 60 = -44$$

Since $D < 0$, the roots are **imaginary**.
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6 Conclusion

- A quadratic equation is of the form $ax^2 + bx + c = 0$.

- The equation can have two roots which may be real or complex.

- Factorization and quadratic formula are the two main methods to solve quadratic equations.

- The discriminant helps determine the nature of roots.