1 Euclid's Division Lemma

Statement: For any two positive integers a and b, where $a > b$, there exist unique integers q (quotient) and r (remainder) such that:

$$a = bq + r, \quad 0 \leq r < b$$

Example: Divide 23 by 5 using Euclid's division lemma.

$$23 = 5 \times 4 + 3$$

Here, $q = 4$ and $r = 3$.
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2 Fundamental Theorem of Arithmetic

Statement: Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order of the factors.
Example:

$$30 = 2 \times 3 \times 5$$

$$84 = 2^2 \times 3 \times 7$$

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3 Prime Factorization

Prime factorization is the process of expressing a number as the product of its prime factors.
Example: Find the prime factorization of 120.

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 2^3 \times 3 \times 5$$

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4 HCF and LCM

4.1 Highest Common Factor (HCF):

The HCF of two or more numbers is the greatest number that divides each of them exactly.

4.2 Least Common Multiple (LCM):

The LCM of two or more numbers is the smallest number that is exactly divisible by each of them.
Example: Find the HCF and LCM of 18 and 24 using the prime factorization method.

$$18 = 2 \times 3^2, \quad 24 = 2^3 \times 3$$

HCF:

$$\text{Common factors: } 2^1 \times 3^1 = 6$$

LCM:

$$\text{All factors: } 2^3 \times 3^2 = 72$$

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5 Relation Between HCF, LCM, and Numbers

The relation between the HCF and LCM of two numbers is given by:

$$\text{HCF} \times \text{LCM} = \text{Product of the Numbers}$$

Example: For numbers 18 and 24:

$$6 \times 72 = 18 \times 24$$

$$432 = 432$$

Thus, the relation is verified.

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6 Decimal Expansion of Rational and Irrational Numbers

6.1 Rational Numbers:

A number is rational if it can be expressed in the form $\frac{p}{q}$, where p and q are integers, and $q \neq 0$. The decimal expansion of rational numbers can be:

- **Terminating:** A decimal that ends after a finite number of digits. Example: $\frac{1}{4} = 0.25$.
- **Non-terminating but Repeating:** A decimal that repeats the same digits infinitely. Example: $\frac{1}{3} = 0.333\ldots$.

6.2 Irrational Numbers:

A number that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers, is an irrational number. Its decimal expansion is:

- Non-terminating and non-repeating. Example: $\pi = 3.141592653\ldots$

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7 How to Prove a Number is Irrational

To prove a number is irrational, we use the method of contradiction.
Example: Prove that $\sqrt{2}$ is irrational.
Proof: Assume $\sqrt{2}$ is rational, so it can be written as:

$$\sqrt{2} = \frac{p}{q}, \quad \text{where } p, q \text{ are coprime integers}$$

Squaring both sides:

$$2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$$

Since p^2 is even, p must also be even. Let $p = 2m$:

$$(2m)^2 = 2q^2 \Rightarrow 4m^2 = 2q^2 \Rightarrow q^2 = 2m^2$$

This implies that q is also even, contradicting the assumption that p and q are coprime. **Thus, $\sqrt{2}$ is irrational.**

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8 Conclusion

- Euclid's division lemma helps in finding HCF.
- The fundamental theorem of arithmetic states that every number can be uniquely factorized into primes.
- The relation between HCF and LCM is given by their product being equal to the product of numbers.
- Decimal expansions help differentiate between rational and irrational numbers.
- The method of contradiction can be used to prove irrationality of numbers.