Solution

MOTION IN A PLANE IMP QUESTIONS

Class 11 - Physics

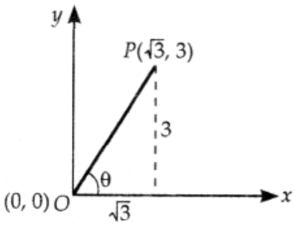
Section A

1.

(c) 60°

Explanation:

The situation is shown in the figure.



If the path OP makes an angle θ with the x-axis, then

$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore \theta$ = 60°

2.

(b) 40 m

Explanation:

Initial velocity u = 54 km/h = 15 m/s

Let Final velocity v = 0

Acceleration a = -0.3 m/s^2

Time taken to stop = t

Using v = u + at

$\Rightarrow$ 0 = 15 + (-0.3)t

$\Rightarrow$ t = 50 sec

It means it has been stopped before 1 minute.

So distance covered in 1 minute is given by

s= ut +1/2 at^2

= 15 $\times$ 60 + 1/2 $\times$ (-0.3) $\times$ (60)^2

= 360 m

Position of locomotive relative to the traffic lights = 400 - 360 = 40 m

3. **(a)** 0, 10 m/s

Explanation:

Average velocity $= \frac{\text{Displacement}}{\text{Time}} = \frac{0}{t}$ = 0

Average speed $= \frac{\text{Distance}}{\text{time}} = \frac{2\pi r}{t}$

$= \frac{2\times3.14\times100}{62.8}$ = 10 ms^{-1}

4.

(d) $2t\sqrt{a^2+b^2}$

Explanation:

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = at^2\hat{i} + bt^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 2at\hat{i} + 2bt\hat{j}$

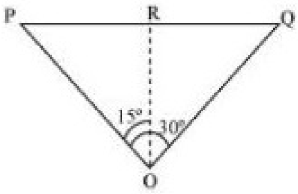
$|\vec{v}| = \sqrt{(2at)^2 + (2bt)^2} = 2t\sqrt{a^2+b^2}$

5. **(a)** 2 m/s
Explanation:
Here r = 20 cm = 0.20 m
ω = 10 rad s^{-1}
$\therefore v = r\omega = 0.20 \times 10$ = 2 ms^{-1}

6.
(d) Equal path lengths are traversed in equal intervals.
Explanation:
In the two-dimensional motion, If instantaneous speed is constant then equal path lengths are traversed in equal intervals of time because speed is a scalar quantity.

7.
(d) $u_x = u_y$ and dx = dy over same time interval dt
Explanation:
Three kinematical equations of motion depend upon values of initial velocity and rate of change of position. The remaining physical quantities such as final velocity, acceleration can be derived using these two.

8.
(b) The acceleration of the particle is necessarily in the plane of motion.
Explanation:
We know that change in acceleration and velocity is in the direction of Force (F) by $\vec{F}$ = m $\vec{a}$ and change in velocity is zero so acceleration will also be zero and will be in the same planes as that of velocity.

9.
(c) 182 m/s
Explanation:
The positions of the observer and the aircraft are shown in the given figure.


Height of the aircraft from ground is given by, OR = 3400 m
Angle subtended between the positions is given by, ∠POQ = 30°
Time = 10 s
In △PRO, we have
tan 15° = $\frac{PR}{OR}$
PR = OR tan 15° = $3400 \times tan 15^\circ$
△PRO is similar to △RQO.
$\therefore$ PR = RQ
PQ = PR + RQ = 2PR = 2 $\times$ 3400 tan 15°
= 6800 $\times$ 0.268 = 1822.4 m
$\therefore$ Speed of the aircraft is given by = $\frac{1822.4}{10}$
= 182.24 m/s $\approx$ 182 m/s

10. **(a)** $5\sqrt{2}$ units
Explanation:
$\vec{v} = \vec{u} + \vec{a}t = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j}) \times 10$
= $5\hat{i} + 5\hat{j}$
$\therefore |\vec{v}| = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ units

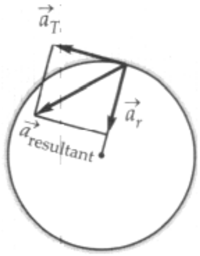
11. **(a)** $\sqrt{\frac{v^4}{r^2}+a^2}$

Explanation:

a_T = a, a_r = a_c = = $\frac{v^2}{r}$

But $a_T \perp a_c$

$\therefore \vec{a}_{\text{resultant}} = \sqrt{a_c^2+a_T^2} = \sqrt{\frac{v^4}{r^2}+a^2}$



12.

(d) parabola

Explanation:

Only in the case of parabolic motion, the direction and magnitude of the velocity changes, acceleration remains the same, and in the case of uniform circular motion, the direction change.

13. **(a)** 2 s, 24 m

Explanation:

u = $u_x\hat{i}$ + $u_y\hat{j}$ = $10\hat{j}$, u_x = 0 and u_y = 10 m/s

a = $a_x\hat{i}$ + $a_y\hat{j}$ = 8i + 2j,

a_x = 8 and a_y = 2 m/s^2

x = u_xt + $\frac{1}{2}a_x t^2$

$\Rightarrow$ 16 = $\frac{8t^2}{2}$

$\Rightarrow$ t = 2 sec

y = u_yt + $\frac{a_y t^2}{2}$ = 10 $\times$ 2 + $\frac{2\times 2^2}{2}$ = 24 m

14.

(d) Decrease

Explanation:

If a ball is thrown upward at an angle, the ball has a vertical and a horizontal component of velocity. The force of gravity is acting on it and its acceleration is in the downward direction. The vertical component of velocity is therefore changing. As the motion and acceleration is in opposite direction so vertical component of velocity decrease.

15. **(a)** sin θ = $\frac{1}{\sqrt{6}}$

Explanation:

Given, u = 60 ms^{-1}

$\therefore$ Maximum height, H = $\frac{u^2 sin^2\theta}{2g}$

$\Rightarrow$ 30 = $\frac{(60)^2 sin^2\theta}{2g}$

$\Rightarrow$ sin$^2\theta$ = $\frac{30 \times 2g}{60 \times 60}$ = $\frac{10}{60}$

$\Rightarrow$ sinθ = $\frac{1}{\sqrt{6}}$

16.

(b) velocity is perpendicular to the direction of acceleration

Explanation:

At the highest point, the projectile has only the horizontal component of velocity while the acceleration due to gravity acts on it vertically downwards.

17.

(b) $\tan^{-1}\frac{1}{2}$

Explanation:

At θ = 45°,

$y=\frac{u^2\sin^2 45^\circ}{2g}=\frac{u^2}{4g}$

$x=\frac{1}{2}\cdot\frac{u^2\sin 90^\circ}{g}=\frac{u^2}{2g}$

$\therefore\tan\beta=\frac{y}{x}=\frac{1}{2}$

$\Rightarrow\beta=\tan^{-1}\left(\frac{1}{2}\right)$

18.

(c) 400 ms^{-1}

Explanation:

$R_{\max}=\frac{u^2}{g}$

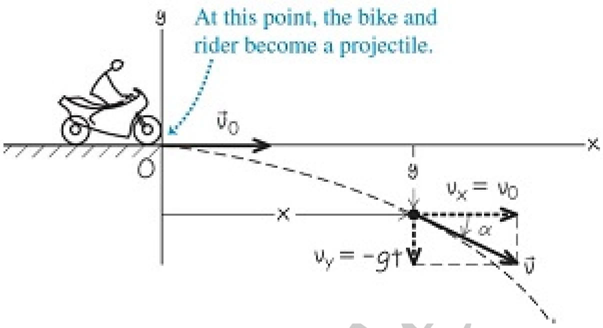
or 16000 m $=\frac{u^2}{10\text{ ms}^{-2}}$

$u=\sqrt{160000}$ = 400 ms^{-1}

19.

(c) 10.2 m/s

Explanation:



the velocity components at t = 0.50 s are

$v_x = v_{ox}$ = 9.0m/s

v_y = −gt = −9.8 × 0.50 = −4.9m/s

The motorcycle has the same horizontal velocity v_x as when it left the cliff at t = 0, but in addition there is a downward (negative) vertical velocity v_y.

The velocity vector at t = 0.50 s is given by ,

$\vec{v}=v_x\hat{i}+v_y\hat{j}=9.0\hat{i}+(-4.9)\hat{j}$

at t = 0.50 s the velocity has magnitude v is given by,

v = $\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(9.0)^2+(-4.9)^2}$

Hence, velocity is v = 10.2 m/s

20.

(b) Horizontal velocity of projectile is constant.

Explanation:

In the case of projectile motion, the vertical component of the particle's velocity changes continuously because of the force acting in the vertical direction which is its own weight. But, in the horizontal direction as there is no force acting on the object. So that its horizontal velocity remains constant.

21.

(c) θ = $\tan^{-1}$(4)

Explanation:

Horizontal range = Maximum height

or $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

or $2 \sin \theta \cos \theta = \frac{\sin^2 \theta}{2}$

or tan θ = 4

$\Rightarrow \theta = \tan^{-1}(4)$

22.

(d) 27°

Explanation:

Horizontal range,

$R = \frac{u^2 \sin 2\theta}{g}$

$\therefore 560 = \frac{82 \times 82 \sin 2\theta}{10}$

$\sin 2\theta = \frac{5600}{6724} = 0.82$

or $2\theta = 53.8^\circ \Rightarrow \theta = 27^\circ$

23.

(b) 605.3 m

Explanation:

For vertical motion of the bomb:

$y = \frac{1}{2} g t^2$

$\Rightarrow 80 = \frac{1}{2} \times 10 \times t^2$

$\therefore$ t = 4s

For horizontal motion of the bomb:

x = ut = 150 $\times$ 4 = 600m

Distance of the target from the dropping point of the bomb

$= \sqrt{x^2 + y^2} = \sqrt{(600)^2 + (80)^2}$ m

= 605.3 m

24.

(d) 40 m

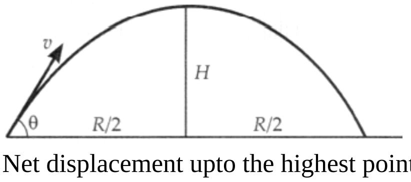
Explanation:

$R_{max} = \frac{u^2}{g} = \frac{20 \times 20}{10}$ = 40 m

25.

(b) $\frac{v}{2} \sqrt{1 + 3 \cos^2 \theta}$

Explanation:



Net displacement upto the highest point

$= \sqrt{H^2 + \left(\frac{R}{2}\right)^2}$

$= \sqrt{\left(\frac{v^2 \sin^2 \theta}{2g}\right)^2 + \left(\frac{v^2 \sin 2\theta}{2g}\right)^2}$

$= \frac{v^2}{2g} \sqrt{\sin^4 \theta + (2 \sin \theta \cos \theta)^2}$

$= \frac{v^2}{2g} \sqrt{\sin^2 \theta \left(\sin^2 \theta + 4 \cos^2 \theta\right)}$

$= \frac{v^2 \sin \theta}{2g} \sqrt{1 + 3 \cos^2 \theta}$

Also, $t=\frac{T_f}{2}=\frac{v\sin\theta}{g}$
$v_{av}=\frac{\text{Net displacement}}{t}=\frac{v}{2}\sqrt{1+3\cos^2\theta}$

26. **(a)** $\frac{20\sqrt{2}}{g}$
Explanation:
$T=\frac{2u\sin\theta}{g}=\frac{2\times20\times\sin45^\circ}{g}=\frac{20\sqrt{2}}{g}$

27.
(d) velocity is minimum
Explanation:
If a projectile is at the highest point of its trajectory, then it has zero vertical.

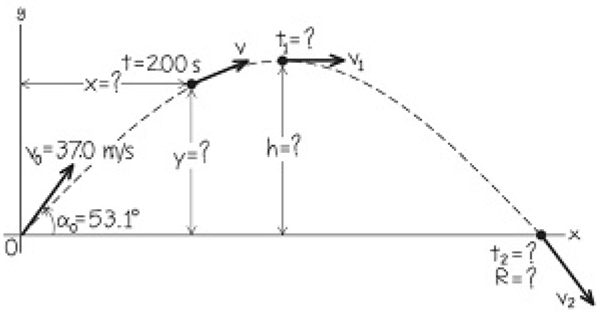
28.
(b) 75 m/s
Explanation:
Initial velocity, u = 150 m/s
Angle $\theta=30^\circ$
Vertical component is given by
$v_y=usin\theta=150sin30^\circ=150\times\frac{1}{2}$
= 75 m/s

29. **(a)** 100 m
Explanation:
projectile is fired at $\theta=15^\circ$, R = 50 m
R = $\frac{u^2\sin2\theta}{g}$
50 = $\frac{u^2\sin2\times15^\circ}{g}\Rightarrow u^2$ = 50 g $\times$ 2
u^2 = 100 g
Now $\theta=45^\circ, u^2$ =100 g
$\therefore R=\frac{u^2\sin2\theta}{g}=\frac{100g\times\sin2\times45^\circ}{g}$
$\Rightarrow$ R = 100 m

30. **(a)** 30°
Explanation:
$\frac{u^2\sin2\theta}{g}=4\sqrt{3}\times\frac{u^2\sin^2\theta}{2g}$
$\Rightarrow 2\sin\theta\cos\theta=2\sqrt{3}\sin^2\theta$
$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=30^\circ$

31.
(d) remains same
Explanation:
Because there is no acceleration or retardation along the horizontal direction, the hence horizontal component of velocity remains same.

32. **(a)** 3.02 s, 44.7 m
Explanation:



The initial velocity of the ball has components

$v_{ox} = v_o \cos \alpha_o = 37.0 \times \cos 53.1^o$

= 22.2 m/s

$v_{oy} = v_o \sin \alpha_o = 37.0 \times \sin 53.1^o$

= 29.6 m/s

At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1; then

$v_y = v_{oy} - gt_1 = 0$

$\Rightarrow t_1 = \frac{v_{oy}}{g} = \frac{29.6}{9.8} = 3.02$ s

The height at the highest point is the value of y at time t_1:

$h = v_{oy}t_1 - \frac{1}{2}g(t_1)^2$

$= 29.6 \times 3.02 - \frac{1}{2} \times 9.8 \times (3.02)^2$

= 44.7 m

33.

(c) 1:3

Explanation:

$H = \frac{u^2 \sin^2 \theta}{2g}$

i.e., $H \propto \sin^2 \theta$

$\therefore \frac{H_1}{H_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$ = 1:3

34. **(a)** 60°

Explanation:

Suppose the particle is projected with velocity u at an angle theta with the horizontal. Horizontal component of its velocity at all height will be u cosθ.

At the greatest height, the vertical component of velocity is zero, so the resultant velocity is

$v_1 = u \cos \theta$

At half the greatest height during upward motion,

y = $\frac{h}{2}$, a_y = -g, $u_y = u \sin \theta$

Using $v_y^2 - u_y^2 = 2a_y y$

We get, $v_y^2 - u^2 \sin^2 \theta = 2(-g)\frac{h}{2}$

or $v_y^2 = u^2 \sin^2 \theta - g \times \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 \theta}{2}$ $\left[\because h = \frac{u^2 \sin^2 \theta}{2g}\right]$

or $v_y = \frac{u \sin \theta}{\sqrt{2}}$

Hence, resultant velocity at half of the greatest height is

$v_2 = \left(\sqrt{v_1^2 + (v_y^2)}\right)$

$= \left(\sqrt{u^2 \cos^2 \theta + \left(\frac{u^2 \sin^2 \theta}{2}\right)}\right)$

Given, $\frac{v_1}{v_2} = \left(\sqrt{\frac{2}{5}}\right)$

$\therefore \frac{v_1^2}{v_2^2} = \frac{u^2 \cos^2 \theta}{u^2 \cos^2 \theta + \left(\frac{u^2 \sin^2 \theta}{2}\right)} = \frac{2}{5}$

or $\frac{1}{1 + \frac{1}{2}\tan^2 \theta} = \frac{2}{5}$

or $2+\tan^2\theta=5$ or $\tan^2\theta=3$
or $\tan\theta=(\sqrt{3})$
$\therefore \theta=60^\circ$

35.

(c) periodic but not simple harmonic

Explanation:

Circular motion with constant speed is periodic but not simple harmonic.

36.

(c) $\frac{v^2}{r}$

Explanation:

For circular acceleration,

$a_c=\frac{v^2}{r}$

37.

(b) 5 m/s^2

Explanation:

a $= rw^2=r\left(\frac{2\pi}{T}\right)^2=\frac{4\pi^2r}{T^2}$

$=\frac{4\pi^2\times5\times10^{-2}}{(0.2\pi)^2}$ = 5 m/s^2

38.

(d) 0.86 ms^{-2}, 54.5^o with the direction of the velocity.

Explanation:

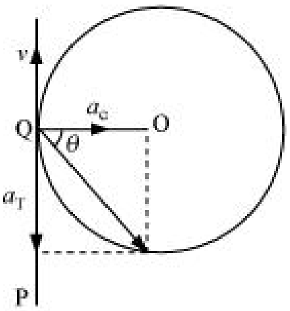
Speed of the cyclist, v = 27 km/h = 7.5 m/s

The radius of the circular turn, r = 80 m

Centripetal acceleration is given as:

$a_c=\frac{v^2}{r}=\frac{(7.5)^2}{80}=$ 0.7 ms^{-2}

The situation is shown in the given figure:



Suppose the cyclist begins cycling from point P and moves toward point Q. At point Q, he applies the breaks and decelerates the speed of the bicycle by 0.5 m/s^2.

This acceleration is along the tangent at Q and opposite to the direction of the motion of the cyclist.

Since the angle between a_c and a_T is 90^0, the resultant acceleration is given by:

a = $\sqrt{(a_c)^2+(a_T)^2}$ = $\sqrt{(0.7)^2+(0.5)^2}$

= $\sqrt{0.74}$ = 0.86 ms^{-2}

tan θ = $\frac{a_c}{a_T}$

where θ is the angle of the resultant with the direction of velocity.

tan θ = $\frac{0.7}{0.5}$= 1.4

θ = $\tan^{-1}(1.4)$ = 54.46^o with the direction of velocity.

39.

(b) 4π rad/s

Explanation:

$\omega = \frac{\theta}{t} = \frac{2\pi \times 120\text{rad}}{60\text{ s}}$ = 4π rad s^{-1}

40. If the magnitude as well as the direction of the acceleration of a body, is constant, it is not necessary that the path of the body is a straight line. For example, in projectile motion, the projectile is under g which has a constant magnitude $\left(9.8\text{m/s}^2\right)$ and constant direction (vertically downward). The path of the projectile is parabolic and not a straight line.
41. A stone cannot be considered as a projectile because a projectile must have two perpendicular components of velocities but in this case a stone has velocity in one direction while going up or coming downwards.

Section B

42. Position,

$\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + 6t\hat{k}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = [4(-\sin 2t)\cdot(2)]\hat{i}$ $+[4(\cos 2t)\cdot(2)]\hat{j} + 6\hat{k}$

= $(-8\sin 2t)\hat{i} + (8\cos 2t)\hat{j} + 6\hat{k}$

Acceleration,

$\vec{a} = \frac{d\vec{v}}{dt}$

= [-8 (cos 2t) (2)] $\hat{i}$ + [8 (-sin 2t) (2)]$\hat{j}$

= (-16 cos 2t) $\hat{i}$ + (-16 sin 2t)$\hat{j}$

When t = $\frac{\pi}{4}$

$\vec{a} = (-16\cos\frac{\pi}{2})\hat{i} + (-16\sin\frac{\pi}{2})\hat{j}$

$= (-16\times 0)\hat{i} + (-16\times 1)\hat{j} = -16\hat{j}$ ms^{-2}

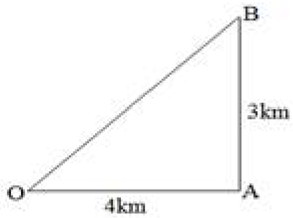
43. Here, straight distance of the object from the radar = OB

$OB = \sqrt{4^2 + 3^2}$

$OB = \sqrt{16 + 9}$

$OB = \sqrt{25} = 5$

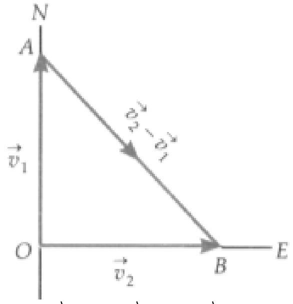
$\Rightarrow$ Range = 5 km



44. In figure,

Initial velocity $\overrightarrow{v_1} = \overrightarrow{OA}$ = 20 ms^{-1}, due north

Findal velocity $\overrightarrow{v_2} = \overrightarrow{OB}$ = 15 ms^{-1}, due east



As $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{v}_2 - \vec{v}_1$

= change in velocity

$\therefore |\vec{v}_2 - \vec{v}_1| = AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{20^2 + 15^2} = \sqrt{625}$ = 25 ms^{-1}

Average acceleration $= \frac{|\vec{v}_2 - \vec{v}_1|}{t} = \frac{25}{50}$ = 0.5 ms^{-2}

45. Radius of the circular path, r = 70 m

Time taken to complete one round, t = 11 s

i. Total length of path

s = $2\pi r = 2 \times \frac{22}{7} \times 70$ = 440 m

ii. As the cyclist returns to the initial position after one round, magnitude of the displacement = 0

iii. Average speed $= \frac{s}{t} = \frac{440\text{ m}}{11\text{ s}}$ = 40 ms^{-1}

iv. Average velocity $= \frac{\text{Displacement}}{\text{Time}} = \frac{0}{11} = 0$

46. i. To the observer inside the train, the ball will appear to move straight vertically upwards and then downwards because with respect to an observer sitting inside the train, the ball has only one velocity acting vertically downward.

ii. To the observer outside the train, the ball will appear to move along the parabolic path because with respect to the observer outside the train, the ball has both horizontal and vertical components of velocity.

47. Clearly, $H = \frac{1}{2}gt^2$

$\therefore t = \sqrt{\frac{2H}{g}}$

Distance from the base of the tower,

x = vt $= v\sqrt{2H/g}$

48. Equating,

$\frac{\mu^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$

$\because \frac{u^2 \sin 2\theta}{g} = R$ (Horizontal Range)

$\sin 2\theta = 1/2 \sin^2\theta$

$\because \frac{u^2 \sin^2\theta}{2g} = h_m$ (Maximum Height)

$2\sin\theta\cos\theta = 1/2\sin^2\theta$

$\tan\theta = 4 \Rightarrow \theta = 75.96^\circ$

49. Let both bodies collide after t sec.

Displacement in the vertical direction by 1st particle = displacement in y-direction by 2nd particle

1st particle is projected with speed u_1 at an angle 60° with horizontal.

After time t sec vertical displacement, S

S = $u_1 \sin 60^\circ t - \frac{1}{2}gt^2$

displacement of 2nd particle after time t sec.

S $= u_2 t - \frac{1}{2}gt^2$

$u_1 \sin 60^\circ t - \frac{1}{2}gt^2 = u_2 t - \frac{1}{2}gt^2$

$u_1 \sin 60^o t = u_2 t$

$\frac{u_1}{u_2} = \frac{1}{\sin 60^\circ}$

$\frac{u_1}{u_2} = \frac{2}{\sqrt{3}}$

50. i. Yes, because the initial vertical velocity of both the balls is zero, and both will cover the same vertical distance under the same vertical acceleration g.

ii. No, because on striking the ground although their vertical velocities will be same but horizontal velocities will be different. Hence their resultant velocities will be different.

51. Given that Horizontal range, R = 3 km and angle of projection of the projectile as 30°

$R = \frac{u^2 \sin 2\theta}{g}$ or $3 = \frac{u^2 \sin 60^\circ}{g} = \frac{u^2}{g}\sqrt{3}/2$

or $\frac{u^2}{g} = 2\sqrt{3}$

Given that the muzzle speed is fixed

Therefore, maximum horizontal range, is

$R_{\max} = \frac{u^2}{g} = 2\sqrt{3} = 3.464\text{km}$

Hence, the bullet cannot hit the target at 5 km.

52. i. As the motion of body is under gravity and no external force acts on the body, the direction of acceleration is always towards the center of earth i.e, downward.

ii. As the ball is thrown vertically upward so its component of horizontal velocity becomes zero. At the highest point, the velocity of the body v_y = 0. Hence, the net velocity of the body at the highest point is zero.

53. Given: $u_y = 0$, $a_y = g = 9.8 m/s^2$

Let the projectile's initial velocity be u.

Time of flight T = 3s (given)

Using $\mathbf{T} = \sqrt{\frac{\mathbf{2h}}{\mathbf{g}}} \Rightarrow \mathbf{h} = \frac{\mathbf{gT^2}}{\mathbf{2}}$

$\therefore \mathbf{h} = \frac{\mathbf{9.8 \times 3^2}}{2}$ = 44.1m

x direction: $a_x = 0 \Rightarrow V_x = u$

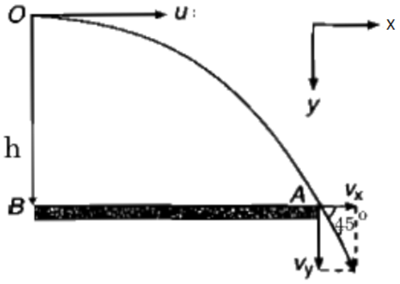
y direction: $V_y = u_y + a_yT$

$\therefore V_y = 0 + 9.8 \times 3 = 29.4$m/s

Also $\frac{\mathbf{V_y}}{\mathbf{V_x}} = \tan 45^o = 1$

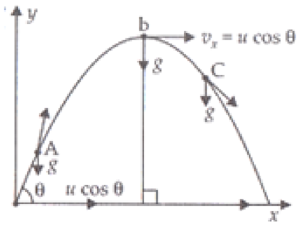
$\Rightarrow V_x = V_y = 29.4$ m/s

Thus initial speed of the projectile $u = V_x = 29.4$ m/s



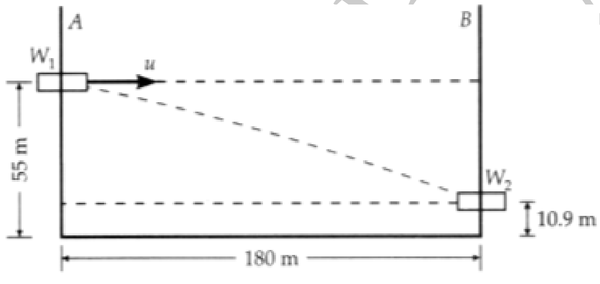
54. The motion of projectile is always parabolic or its part. Its velocity at any point of its path is always tangentially toward the direction of motion so velocities at points A, B and C are tangentially shown,

The point B is at its maximum height of trajectory. So the vertical component of B $v_y = 0$ and horizontal component is u cosθ.



As the direction of acceleration is always in the direction of the force acting on it. The gravitational force is acting on the body hence the direction of acceleration is always vertically downward equal to acceleration to gravity (g).

55. In the figure, A and B are two tall buildings that are 180 m apart. W_1 and W_2 are the two windows in A and B respectively.



Vertical downward distance to be covered by the ball,

= Height of W_1 - Height of W_2

= 55 - 10.9 = 44.1 m

Initial vertical velocity of ball,

$u_y = 0$

As $y = u_yt + \frac{1}{2}gt^2$

$\therefore 44.1 = 0 + \frac{1}{2} \times 9.8t^2$

Required horizontal velocity

$= \frac{\text{Horizontal distance}}{\text{Time}} = \frac{180 \text{ m}}{3 \text{ s}}$ = 60 ms^{-1}

56. Velocity of projectile = u = 147 ms^{-1} angle of projection α = 60°

Let the time taken by a projectile from O to A, be t and its velocity is v at point A where the direction is β = 45°.

As a horizontal component of velocity remains constant during the projectile motion.

$\Rightarrow$ v cos 45^o = u cos 60^o

$\Rightarrow \mathbf{v} \times \frac{1}{\sqrt{2}} = 147 \times \frac{1}{2} \Rightarrow \mathbf{v} = \frac{147}{\sqrt{2}}$ ms^{-1}

For vertical motion,

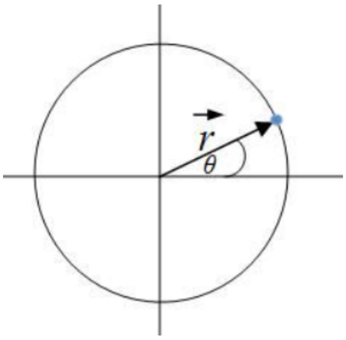
$v_y = u_y - gt$

$\Rightarrow$ v sin 45^o = 45 sin60^o - 9.8t

$\Rightarrow \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 147 \times \frac{\sqrt{3}}{2}$ - 9.8 t

9.8 t = $\frac{147}{2}(\sqrt{3} - 1)$

t = 5.49 s

57. 

Let the position vector of the particle in circular motion be

$\vec{r} = r(\hat{i}\cos(\theta) + \hat{j}\sin(\theta))$

Differentiating both sides w.r.t. t

$\frac{d\vec{r}}{dt} = r(-\hat{i}\sin(\theta) + \hat{j}\cos(\theta))\frac{d\theta}{dt}$

$\Rightarrow \vec{v} = \omega r(-\hat{i}\sin(\theta) + \hat{j}\cos(\theta))$

or, v = ωr

Differentiating both sides w.r.t. t

$\frac{d\vec{v}}{dt} = -\omega^2 r(\hat{i}\cos(\theta) + \hat{j}\sin(\theta))$

$\Rightarrow \vec{a}_c = -\omega^2 r(\hat{i}\cos(\theta) + \hat{j}\sin(\theta))$

$\Rightarrow \vec{a}_c = -\omega^2 \vec{r}$

This shows the centripetal acceleration acts in a direction opposite to $\vec{r}$, that is along its centre

$\Rightarrow |\vec{a}_c| = \omega^2 r$

Again we know

v = rω

thus

$a_c = \frac{v^2}{r}$

58. When an object moves in a circular path with constant speed then the motion is called **uniform circular motion.**

Time period - The time taken by the object to complete one revolution

Frequency - The total number of revolutions in one second is called the frequency.

Angular velocity - It is defined as the time rate of change of angular displacement.

$W = \frac{2\pi}{T} = 2\pi\nu \quad \left(\because \frac{1}{T} = \nu\right)$

59. Here, radius of the horizontal circular loop traversed by the aeroplane r = 1 km = 1000 m,

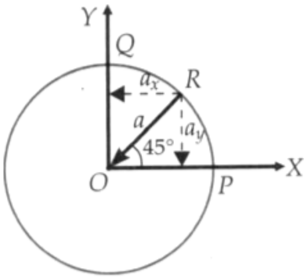
with constant speed v = 900 km h^{-1} = $\frac{900 \times (1000\text{m})}{(60 \times 60\text{s})}$= 250 ms^{-1}

We know that Centripetal acceleration, $a = \frac{v^2}{r} = \frac{(250)^2}{1000}$

Now, $\frac{a}{g} = \frac{(250)^2}{1000} \times \frac{1}{9.8} = 6.38$

Hence a_c is 6.4 times more than the g in this case.

60. As shown in figure (b), let R be the midpoint of arc PQ. Then $\angle POR = 45°$.



(b)

The magnitude of acceleration at R,

$a = \frac{v^2}{r} = \frac{(2)^2}{4}$ = 1 cms^{-2}

The acceleration acts along with RO.

The magnitude of component of an along X-axis,

$a_x = a \cos 45^o = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ cms^{-2}

$\therefore \hat{a}_x = -\frac{1}{\sqrt{2}}\hat{x}$

The magnitude of the component of an along Y-axis,

$a_y = 1 \times \sin 45^\circ = \frac{1}{\sqrt{2}}$ cm^{-2}

$\therefore \hat{a}_y = -\frac{1}{\sqrt{2}}\hat{y}$

Hence $\hat{a} = \hat{a}_x + \hat{a}_y = -\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$

61. In hour hand of a watch (T) = 12h

$W_H = \frac{2\pi}{12}$

For rotation of earth T = 24h

$We = \frac{2\pi}{24}$

$\Rightarrow W_H : We = \frac{24}{12} = 2$

$W_H = 2W_e$

62. The distance 's' covered by a body travelling in an arc of radius Y and turning its radial line by $'\theta'$ is given by

$s = r\theta$

Differentiating both sides w.r.t. time, we have

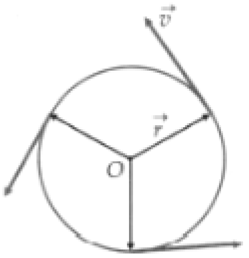
$\frac{ds}{dt} = \mathbf{r}\frac{d\theta}{dt}$

i.e., $v = r\omega$

or Linear velocity = radius $\times$ angular velocity

Thus, the direction of velocity is always along the tangent at any point in the circular path.

63. Uniform circular motion is an accelerated motion. In uniform circular motion, the speed of the body remains the same but the direction of motion changes at every point. Figure shows the different velocity vectors at different positions of the particle. At each position, the velocity vector $\vec{v}$ is perpendicular to the radius vector $\vec{r}$. Thus the velocity of the body changes continuously due to the continuous change in the direction of motion of the body. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.



Direction of velocity changes at every point.

64. No, a physical quantity having both magnitude and direction need not be considered a vector. For example, despite having magnitude and direction, the current is a scalar quantity. The essential requirement for a physical quantity to be considered a vector is that it should follow the law of vector addition.

No, generally speaking, the rotation of a body about an axis is not a vector quantity as it does not follow the law of vector addition. However, rotation by a certain small angle follows the law of vector addition and is therefore considered a vector.

Section C

65. a. Total distance travelled = 23 km

Total times taken = 28 min $= \frac{28}{60}h$

Average speed of the taxi $= \frac{\text{total dis tan}\, ce \text{ travelled}}{\text{time interval}}$

$s_{av} = \frac{23}{\frac{28}{60}} = 49.29\text{km/h}$

b. Distance between the hotel and the station = 10 km = Displacement of the car

Average velocity $= \frac{\text{displacement}}{\text{time interval}} = \frac{10}{\frac{28}{60}} = 21.43\text{km}/h$

Therefore, the two physical quantities (average speed and average velocity) are not equal.

66. Yes, for the longest jump the player should throw himself at an angle of 45° with respect to horizontal. The vertical height for this angle is

H $= \frac{u^2 sin^2 45^o}{2g} = \frac{u^2}{4g}$

where u is the velocity of projection. If the vertical height is different from $\frac{u^2}{4g}$, then the angle will be different from 45° and the horizontal distance covered will also be less.

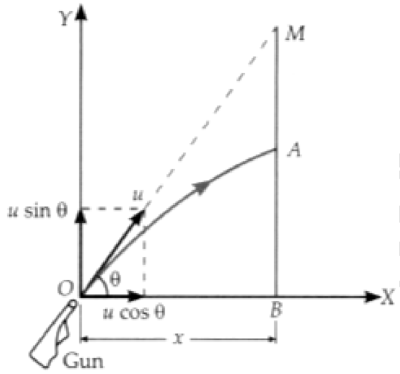
Clearly, the span of jump depends upon

i. the initial velocity of the jump,
ii. the angle of projection

67. As shown in Figure, the gun at O is directed towards the monkey at position M. Suppose the bullet leaves the barrel of the gun with velocity u at an angle θ with the horizontal. Let the bullet cross the vertical line MB at A after time t. Horizontal distance travelled,

OB = x = $u \cos\theta \cdot t$

or $t = \frac{x}{u \cos\theta}$ ..... (i)



For motion of the bullet from O to B, the vertical range is

AB $= u \sin\theta \cdot t - \frac{1}{2}gt^2 = u \sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}gt^2$

$= x\tan\theta - \frac{1}{2}gt^2$ [using (i)]

Also MB = x tan θ

$\therefore$ MA = MB - AB

$= x\tan\theta - \left[x\tan\theta - \frac{1}{2}gt^2\right] = \frac{1}{2}gt^2$

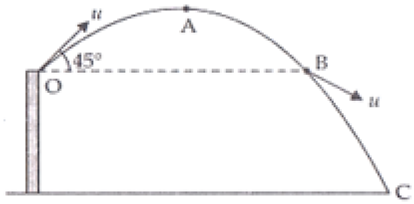
Thus in a time t the bullet passes through A a vertical distance $\frac{1}{2}gt^2$ below M.

The vertical distance through which the monkey falls in time t

$= 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

Thus the bullet and the monkey will always reach the point A at the same time. Hence the bullet will always hit the monkey whatever be the velocity of the bullet.

68. A ball is projected from O at an angle of 45^{o} with horizontal. From O to A body rises up, height increases so its speed & hence KE (speed) decreases. From A to B it’s speed again increases as its height decreases and become equal to its’s initial speed at O, because O and B are on the same horizontal line.



From B to C, its height again decreases so its speed from B to C increases and become maximum at C

$v_y = v_x = u\cos(45) = \frac{u}{\sqrt{2}} m/s$ at Point B.

Now for motion along BC, $v'_x = \frac{u}{\sqrt{2}}$ remains constant but

$v'_y = v_y + gt$ thus velocity along y - axis changes.

net velocity $v = \sqrt{(v'_x)^2 + (v'_y)^2}$ become maximum at point C

$v = \sqrt{\frac{u^2}{2} + \frac{u^2}{2} + g^2t^2}$

$v = \sqrt{u^2 + g^2t^2}$

Hence,

i. Greatest speed of ball is at C .
ii. The smallest speed will be at A . Where at maximum height and $v_y = 0$ and has the only horizontal speed of constant value $u_x = \frac{u}{\sqrt{2}}$.
iii. For motion between O to A, acceleration a = -g and for A to C acceleration a = +g and is constant during AC.

69. If the two projectiles are thrown with velocities u_1 and u_2 at angle θ_1 and θ_2 with horizontal, then their maximum heights will be

$H_1 = \frac{u_1^2 \sin^2\theta_1}{2g}$ and $H_2 = \frac{u_2^2 \sin^2\theta_2}{2g}$

But $H_1 = H_2$

$\therefore \frac{u_1^2 \sin^2\theta_1}{2g} = \frac{u_2^2 \sin^2\theta_2}{2g}$

or $u_1 \sin\theta_1 = u_2 \sin\theta_2$ ..... (i)

Times of flight for the two projectiles are

$T_1 = \frac{2u_1 \sin\theta_1}{g}$ and $T_2 = \frac{2u_2 \sin\theta_2}{g}$

Making use of equation (i), we get

$T_1 = T_2 = \frac{2u_1 \sin\theta_1}{g} = \frac{2u_2 \sin\theta_2}{g}$

Times taken to reach the highest point in the two cases will be

$t_1 = \frac{u_1 \sin\theta_1}{g}$ and $t_2 = \frac{u_2 \sin\theta_2}{g}$

$\therefore t_1 + t_2 = \frac{u_1 \sin\theta_1}{g} + \frac{u_2 \sin\theta_2}{g}$

$= \frac{2u_1 \sin\theta_1}{g}$ or $\frac{2u_2 \sin\theta_2}{g}$ [using (i)]

or [$t_1 + t_2$ = Time of flight of either projectile.

70. Let u be the muzzle speed of the bullet fired from the gun (on the top of the tower) at an angle θ with the horizontal, as shown in Figure.

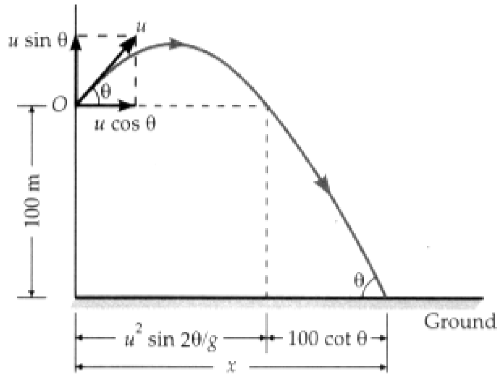
Clearly, the total range of firing on the ground is

$x = \frac{u^2 \sin 2\theta}{g} + 100\cot\theta$

$\therefore \quad \frac{dx}{d\theta} = \frac{u^2 \times 2\cos 2\theta}{g} + 100 \times \left(-\operatorname{cosec}^2\theta\right)$

$= \frac{2u^2}{g}\left(1 - 2\sin^2\theta\right) - \frac{100}{\sin^2\theta}$

$= 4500 - 9000\sin^2\theta - \frac{100}{\sin^2\theta}$



For x to be maximum,

$\frac{dx}{d\theta} = 0$

4500 - 9000 $\sin^2\theta - \frac{100}{\sin^2\theta} = 0$

$90\sin^4\theta - 45\sin^2\theta + 1 = 0$

or $\sin^2\theta = \frac{45 \pm \sqrt{(-45)^2 - 4\times 90\times 1}}{2\times 90}$

$= \frac{45\pm 40.80}{180}$

Taking only positive sign,

$\sin^2\theta$ = 0.4767

or $\sin\theta$ = 0.6904

or θ = 43.7°

71. Given: Horizontal Range, R = 3 km = 3000 m

Angle of projection, θ = 30°

Acceleration due to gravity, $g = 9.8 \text{m/s}^2$

Horizontal range for the projection velocity u_0, is given by the relation:

$R = \frac{u^2 \sin 2\theta}{g}$

$3000 = \frac{u_0^2}{g}\sin 60^\circ$

$3000 = \frac{u_0^2}{g} \times \frac{\sqrt{3}}{2}$

$\frac{u_0^2}{g} = 2\sqrt{3}\times 1000$ ....(i)

The maximum range (R_{max} is achieved by the bullet when it is fired at an angle of 45° with the horizontal)

$R_{max} = \frac{u_0^2}{g}$ ....(ii)

On comparing equations (i) and (ii), we get:

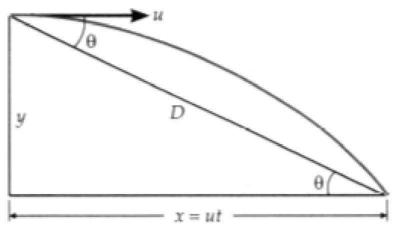
$R_{max} = 2\sqrt{3}\ \times 1000 = 2\times 1.732\times 1000 = 3.46\ km$

Hence by keeping the same muzzle speed u, one can not hit a target which is 5 km away just by changing projection angle.

72. In the baseball game, a player has to throw a ball so that it goes a certain distance in the minimum time. The time would depend on velocity of ball and angle of throw with the horizontal. Thus, while playing a baseball game, the speed and angle of projection have to be adjusted suitably so that the ball covers the desired distance in minimum time. So, a player has to see the distance and air resistance while playing with a baseball game.

73. The horizontal distance covered in time t,

x = ut or $t = \frac{x}{u}$ ..... (i)



The vertical distance covered in time t,

$y = 0 + \frac{1}{2}gt^2 = \frac{1}{2}g\times\frac{x^2}{u^2}$ [using (i)]

Also $\frac{y}{x} = \tan\theta$ or y = x tan θ

$\therefore \frac{gx^2}{2u^2} = x\tan\theta$

or $x\left(\frac{gx}{2u^2}-\tan\theta\right)=0$

As x = 0 is not possible, so $x=\frac{2u^2\tan\theta}{g}$

The distance of the point of strike from the point of projection is

$D=\sqrt{x^2+y^2}=\sqrt{x^2+(x\tan\theta)^2}$

$=x\sqrt{1+\tan^2\theta}=x\sec\theta$

or $D=\frac{2u^2}{g}\tan\theta\sec\theta$

74. Let θ and (90 - θ) be the angles of projection for a projectile thrown with same initial velocity u.

Range of projectile = $\frac{u^2\sin^2\theta}{g}$

for angle θ R_1 = $\frac{u^2\sin^2\theta}{g}$ ...(1)

for angle (90 - θ), R_2 = $\frac{u^2\sin 2(90-\theta)}{g}$ = $\frac{u^2\sin(180-2\theta)}{g}$ = $\frac{u^2\sin 2\theta}{g}$

R_2 = $\frac{u^2\sin 2\theta}{g}$ ...(2)

from (1) and (2), we get $R_1 = R_2$

Maximum height for θ angle of projection is

H_1 = $\frac{u^2\sin^2\theta}{2g}$

Maximum height for (90 - θ) angle of projection is

H_2 = $\frac{u^2\sin^2(90-\theta)}{2g}$ = $\frac{u^2\cos^2\theta}{2g}$

for θ = 30^o

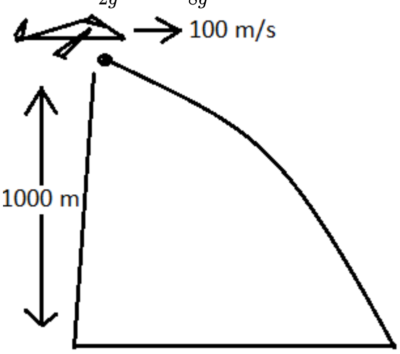
H_1 = $\frac{u^2\sin^2 30}{2g}$ = $\frac{u^2}{8g}$

H_2 = $\frac{u^2\cos^2 30}{2g}$ = $\frac{3u^2}{8g}$

for θ = 60^o

H_1 = $\frac{u^2\sin^2 60}{2g}$ = $\frac{3u^2}{8g}$

H_2 = $\frac{u^2\cos^2 60}{2g}$ = $\frac{u^2}{8g}$

75. 

i. Time Taken;

$T=\sqrt{\frac{2H}{g}}=\sqrt{\frac{2\times1000}{10}}$

= $10\sqrt{2}$sec

ii. When bomb hits the target, it will have 2 components,

v_x and v_y

v_x = 100m/s ($\because$ a_x = 0)

Now, Time of Height

$T=\sqrt{\frac{2H}{g}}=\sqrt{\frac{2\times1000}{10}}$ $=\sqrt{200}$ sec $=10\sqrt{2}sec$

To find V_y

In vertical direction, u_y = 0, $t=10\sqrt{2}$, a = -g s = -1000m

$\therefore v_y^2-v_y^2=2a_ys_y$

v_y^2 = 2(-10)(-1000)

v_y = $100\sqrt{2}$ m/s

$v_{nef} = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(100)^2 + (100\sqrt{2})^2} = 100\sqrt{3}$ m/s

iii. Range = (TOF) $\times$ U_x

$= 10\sqrt{2} \times 100 = 1000\sqrt{2}\,\text{m}$