Solution

MOTION IN A STRAIGHT LINE IMP QUESTIONS

Class 11 - Physics

Section A

1.

(b) $\frac{\alpha\beta t}{\alpha+\beta}$

Explanation:

Initial velocity (u) = 0; Acceleration in the first phase = α

Deceleration in the second phase = β and total time = t

When car is accelerating then final velocity (v) = u + at = 0 + at_1

or $t_1 = \frac{v}{\alpha}$ and when car is decelerating,

then final velocity 0 = v − βt or $t_2 = \frac{v}{\beta}$.

Therefore total time (t) = $t_1 + t_2 = \frac{v}{\alpha} + \frac{v}{\beta}$

t = $v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = v\left(\frac{\beta+\alpha}{\alpha\beta}\right)$ or v = $\frac{\alpha\beta t}{\alpha+\beta}$

2.

(c) 10 m

Explanation:

Let maximum height attained by the ball be H.

Using $v^2 = u^2 - 2gh$

At A,$(10)^2 = u^2 - 2 \times 10 \times \frac{H}{2}$

$\Rightarrow u^2 = 100 + 10H$ …(i)

At O', $(0)^2 = u^2 - 2 \times 10 \times H$

$\Rightarrow u^2 = 20H$ …(ii)

20H = 100 + 10H

$\Rightarrow$ 10H = 100

$\therefore$ H = 10m

3.

(c) 56 m

Explanation:

Distance travelled by the particle is $x = 40 + 12t - t^3$

We know that velocity is rate of change of distance i.e., v = $\frac{dx}{dt}$

$\therefore v = \frac{d}{dt}(40 + 12t - t^3) = 0 + 12 - 3t^2$

but final velocity v = 0

$12 = 3t^2 = 0$ or $t^2 = \frac{12}{3} = 4$

or t = 2s

Hence, distance travelled by the particle before coming to rest is given by

$x = 40 + 12(2) - (2)^3 = 56$m

4.

(d) -9 m/s

Explanation:

$s = t^3 - 6t^2 + 3t + 4$

$v = \frac{ds}{dt} = 3t^2 - 12t + 3$

a = 6t - 12 = 0

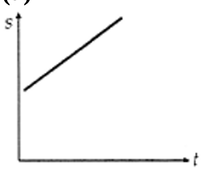
or t = 2 s
At t = 2s,
v = 3 × 2^2 - 12 × 2 + 3 = -9 m/s

5.
(d) straight line
Explanation:
As acceleration (g) remains constant throughout, the velocity-time graph is a straight line.

6.
(b) $\sqrt{3}v_0$
Explanation:
$0^2 - v_0^2$ = 2(-g)h
or v_0^2 = 2gh i.e., $v_0^2 \propto h$
$\therefore v_0'^2 \propto 3h$
$\therefore \frac{v_0'^2}{v_0^2} = \frac{3h}{h} = 3$
or $v_0' = \sqrt{3}v_0$

7.
(c) $\sqrt{t_1 t_2}$
Explanation:
For upward motion,
h = -vt_1 + $\frac{1}{2}gt_1^2$
For downward motion,
h = vt_2 + $\frac{1}{2}gt_2^2$
Now, $\frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g$ (t_1 + t_2) or h = $\frac{1}{2}$gt_1t_2
Also, h = $\frac{1}{2}$gt^2
$\therefore \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2$ or t = $\sqrt{t_1t_2}$

8. **(a)** 36.4 km
Explanation:
distance S_1 travelled in 1 minute = ut+1/2 at^2 =
$x = \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times 60^2 = 18000\ m$
distance attained after 1 min,
or s_2 = $\frac{(600)^2}{2\times9.8}$ = 18367.3 m
Maximum height reached
= s_1 + s_2 = 18000+18367.3 = 36367.3 m = 36.4 km

9. **(a)**


Explanation:
As in uniform motion, the distance-time graph would be a straight line, because the equal distance is covered in equal units of time.

10.
(c) 7 : 9

Explanation:

Distance covered in n^{th} second is given by

$s_n = u + \frac{a}{2}(2n - 1)$

Given: u = 0, a = g

$\therefore s_4 = \frac{g}{2}(2 \times 4 - 1) = \frac{7g}{2}$

$s_5 = \frac{g}{2}(2 \times 5 - 1) = \frac{9g}{2}$

$\therefore \frac{s_4}{s_5} = \frac{7}{9}$

11.

(d) 88 s

Explanation:

Total distance = Length of train + Length of bridge

= (100 + 1000)m = 1100 m

Speed = 45 km/h = $45 \times \frac{5}{18}$ m/s = $\frac{25}{2}$ m/s

Time taken = $\frac{\text{Total distance}}{\text{Speed}}$

= $\frac{1100}{\frac{25}{2}}$ s = 88 s

12. **(a)** the acceleration is independent of the velocity

Explanation:

Velocity and acceleration are independent of each other.

13.

(c) $\frac{2V_1V_2}{V_1+V_2}$

Explanation:

Time t_1 taken in half distance $t_1 = \frac{L}{v_1}$

Time t_2 taken in half dsitance $t_2 = \frac{L}{v_2}$

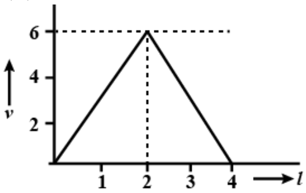
Total time (t) taken in distance (L + L) = $\frac{L}{w_1} + \frac{L}{v_2} = \frac{L(v_2+v_1)}{v_1v_2}$

Total distance = L + L = 2L

$\therefore$ Average speed $v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2L}{\frac{L(v_2+v_1)}{v_1v_2}} = \frac{2v_1v_2}{(v_1+v_2)}$

14.

(b)

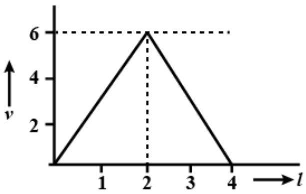


Explanation:

Area under a - t curve = Change in velocity

For constant acceleration, $v \propto t$

As a is first +ve and then -ve, so the correct v - t graph is the one given in option.



15.

(d) 8 m

Explanation:

x = $(t - 2)^2$

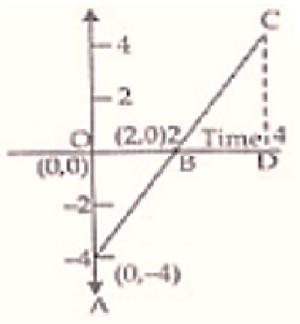
v = $\frac{dx}{dt}$ = 2(t - 2)m/s

a = $\frac{d^2}{dt^2}$ = 2(1 - 0) = 2 ms^{-2}

at t = 0, v_o = 2(0 - 2) = -4 m/s

at t = 2, v_2 = 2(2 - 2) = 0 m/s

at t = 3, v_4 = 2(4 - 2) = 4 m/s



Distance = Area between time axis and(v - t) graph

= area ΔOAB + area ΔBCD

= $\frac{1}{2} \times OB \times OA + \frac{1}{2} \times BD \times CD$

= $\frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2 \times 4$

Distance = 8 m

16. **(a)** 6 ms^{-1}

Explanation:

Let v be the full speed of the man. He catches the bus when his speed becomes equal to the speed of the bus.

Distance = Velocity $\times$ time

6 + x = vt

x = Distance travelled by the bus

v = Velocity after time t

s = ut + $\frac{1}{2}$$at^2$

x = 0 + $\frac{1}{2} \times 3t^2 = \frac{3}{2}t^2$

For the bus, $v^2 - u^2 = 2as$

$v^2 = 2 \times 3 \times x = 6x$

$\therefore (6 + x)^2 = v^2t^2 = 6x \times \frac{2}{3}x = 4x^2$

$\Rightarrow$ 6 + x = $\pm$2x

$\Rightarrow$ x = 6m x $\neq$ - 2

v = $\sqrt{6x} = \sqrt{6 \times 6}$ = 6 ms^{-1}

17. **(a)** 2

Explanation:

Initial velocity, u = 0 m/s

final velocity = v

Time t = 2 s

Acceleration, a = 1 m/s^2

We know, v = u + at

$\Rightarrow v = 0 + 1 \times 2$

$\Rightarrow$ v = 2 m/s

18. **(a)** $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

Explanation:

For a particle released from a certain height the distance covered by the particle in relation with time is given by, $h = \frac{1}{2}gt^2$

For first 5sec, $h_1 = \frac{1}{2}g(5)^2 = 125$

Further next 5 sec, $h_1 + h_2 = \frac{1}{2}g(10)^2 = 500$

$\Rightarrow h_2 = 375$

$h_1 + h_2 + h_3 = \frac{1}{2}g(15)^2 = 1125$

$\Rightarrow h_3 = 625$

$h_1 = 3h_1$, $h_3 = 5h_1$

or $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

19.

(c) 2.5 s

Explanation:

Let the two balls meet after time t. Then the distances covered by the two balls will be

$s_1 = \frac{1}{2}gt^2$

and $s_2 = ut - \frac{1}{2}gt^2$

But $s_1 + s_2 = \frac{1}{2}gt^2 + ut\frac{1}{2}gt^2 = 100$ m

$\therefore ut = 100$

$\Rightarrow t = \frac{100}{u} = \frac{100}{40} = 2.5$ s

20.

(c) 6 m/s

Explanation:

Here u = 10 m/s, t = 3s, v = 16 m/s

$\therefore a = \frac{v-u}{t} = \frac{16-10}{3} = 2$ m/s^2

Velocity at 2s before the given instant,

$10 = u + 2 \times 2$ [v = u + at]

u = 10 - 4 = 6 m/s

21.

(d) 31.0 s

Explanation:

The initial speed of the car u = 45 m/s

Let t = 0 s when the car passes the trooper.

The trooper starts from rest 1 s after the car passes the billboard.

In this 1 s the car would have covered a distance of 45 m.

Let y be the time at which the trooper overtakes the car.

Distance covered by car at time y

S = 45 + uy = 45 + 45y......(1)

(The distance is measured from the billboard)

The same distance is covered by the trooper also.

$S = 0 + \frac{1}{2} \times 3 \times (y)^2$....(2)

(Initial speed of trooper = 0).

Equating (1) and (2)

$45 + 45y = \frac{3}{2}(y)^2$

$\Rightarrow 3y^2 - 90y - 90 = 0$

$\Rightarrow y^2 - 30y - 30 = 0$

Using quadratic formula, we get:

$\Rightarrow \frac{30 \pm \sqrt{900+120}}{2} = \frac{30 \pm 31.93}{2} = 30.97 \text{ s} \approx 31 \text{ s}$

neglected negative value of y.

22.

(c) 45 m

Explanation:

S = $\frac{1}{2}$ $\times$ 10 $\times$ t^2 and (S - 400) = $\frac{1}{2}$ $\times$ 10(t - 2)^2

$\therefore$ S − (S − 400) = 5T^2 − 5(t − 2)^2

or 40 = 20t − 20 or t = 3s

$\therefore$ S = $\frac{1}{2}$ $\times$ $\times$ 10 $\times$ 3^2 = 45m

23.

(b) 16.8 km/h

Explanation:

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

= $\frac{s_1 + s_2}{t_1 + t_2} = \frac{8.4 \text{ km} + 2 \text{ km}}{\frac{8.4}{70} \text{ h} + \frac{30}{60} \text{ h}}$

= $\frac{10.4}{0.12 + 0.5} = \frac{10.4}{0.62}$

$\simeq$ 16.8 km/h

24.

(d) average velocity

Explanation:

Average velocity is the displacement of an object, divided by the time it took to cover that distance.

$V_{average} = \frac{\Delta x}{\Delta t}$

Displacement is the straight line distance between the starting point and ending point of an object's motion.

Velocity is referred to as a vector quantity because it has both magnitude and direction.

25.

(d) 63.0

Explanation:

Initial velocity u = 63 m/s

At it stops final velocity v = 0 m/s

Time taken t = 2 s

v = u+ at

0 = 63 + 2a

a = -31.5 m/s^2

We know,

$s = ut + \frac{1}{2}at^2$

s = 63 $\times$2 + 1/2 $\times$ (-31.5)$\times$ 2^2

s = 63 m

26.

(c) -31.5

Explanation:

Initial velocity, u = 63 m/s

As it stops, so final velocity, v = 0 m/s

Time t = 2.0 s

We know that, v - u = at

|$\Rightarrow a = \frac{v-u}{t}$

$\Rightarrow a = \frac{0-63}{2}$

$\Rightarrow a = -31.5\ m/s^2$

27.

(c) 20 m

Explanation:

u_1 = 40 m/s, u_2 = 20 m/s

a_1 = -4 m/s^2, a_2 = -$\frac{10}{4}$ m/s^2

Let s_1, s_2 be the distance travelled by train I and train II before halting.

$\therefore s_1 = \frac{-u_1^2}{2a_1}$; s2 = $\frac{-u_2^2}{2a_2}$

s_1 = $\frac{1600}{8}$ = 200 m

s_2 = $\frac{400}{5}$ = 80 m

∴ Separation between the trains when both have stopped is,

s - s_1 - s_2 = 300 - 200 - 80 = 20 m

28. **(a)** frame of reference consisting of a clock and a Cartesian system having three mutually ⊥ axes, (X,Y, and Z)

Explanation:

Motion is a change in position of an object with time. In order to specify the position, we need to use a reference point and a set of axes. It is convenient to choose a rectangular coordinate system consisting of three mutually perpendicular axes, labelled X-, Y-, and Z- axes.

The point of intersection of these three axes is called origin (O) and serves as the reference point. The coordinates (x, y. z) of an object describe the position of the object with respect to this coordinate system.

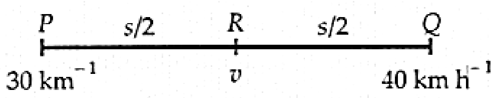
To measure time, we position a clock in this system. This coordinate system along with a clock constitutes a frame of reference.

29.

(d) $25\sqrt{2}$ Km/h

Explanation:

The situation is shown below:



$(40)^2 - (30)^2$ = 2as

or a = $\frac{(40)^2-(30)^2}{2s} = \frac{350}{s}$ ...(i)

Also, $v^2 - (30)^2$ = 2a$\frac{s}{2}$

or $v^2 - (30)^2$ = 2× $\frac{350}{s} \times \frac{s}{2}$ [using(i)]

or v = $25\sqrt{2}$ Km/h

30.

(d) 2 sec

Explanation:

Suppose the two balls cross each other after time t. Then the distances covered by the two balls will be

s_1 = $\frac{1}{2}$gt^2

and s_2 = ut - $\frac{1}{2}$gt^2

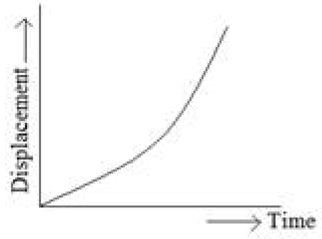
But s_1 + s_2 = $\frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2$ = 100m

∴ ut = 100

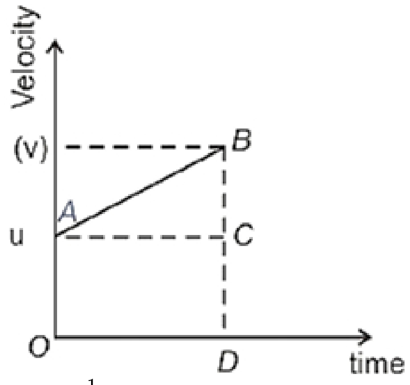
or t = $\frac{100}{u} = \frac{100}{50}$ = 2s

Section B

31. The displacement-time graph for uniformly accelerated motion is shown below. The graph is parabolic in shape.



32.



Area = $\frac{1}{2}$(OA + BD) $\times$ OD

= $\frac{1}{2}$(u + v) $\times$ t

= v = u + at

= $\frac{1}{2}$(u + u + at) $\times$ t

= ut + $\frac{1}{2}$ at^2 = Displacement

33. Here u = 2 ms^{-1}, g = -9.8 ms^{-2}, t = 2 s

i. v = u + gt = 2 - 9.8 $\times$ 2 = -17.6 ms^{-1}.
A negative sign shows that the velocity is directed vertically downwards.

ii. Distance covered by the food packet in 2 s,
s = ut + $\frac{1}{2}at^2$ = 2 $\times$ 2 - $\frac{1}{2}$ $\times$ 9.8 $\times$ 2^2
= 4 - 19.6 = - 15.6 m
Thus the food packet falls through a distance of 15.6 m in 2 s but in the meantime, the helicopter rises up through a distance
= 2 ms^{-1} $\times$ 2 s = 4 m
$\therefore$ After 2 s, the distance of the food packet from the helicopter
= 15.6 + 4 = 19.6 m

34. In time 0 to 2.5 s, acceleration of the vehicle,

a = Slope of OA = $\frac{20-0}{2.5-0}$ = 8 ms^{-2}

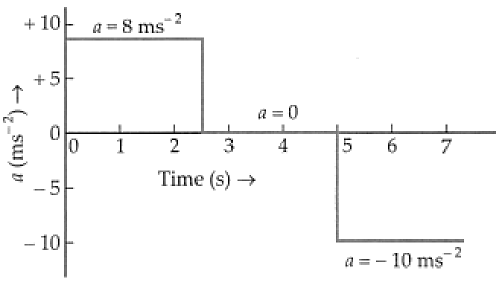
In time 2.5 to 5 s, the velocity of the vehicle is uniform.

So acceleration = 0.

In time 5 to 7 s, acceleration of the vehicle,

a = Slope of BC = $\frac{0-20}{7-5}$ = -10 ms^{-2}

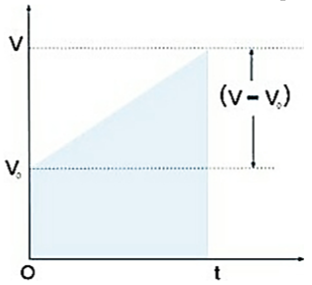
Hence the acceleration-time graph will be as shown in Fig.



35. i. The carriage can be considered as the point object because the distance between two stations is much larger than the size of the carriage.

ii. The monkey can be considered as a point object because its size is much smaller than the distance covered by the cyclist.

iii. The spinning ball cannot be considered as point object because its size is quite appreciable as compared to the distance through which it turns on hitting the ground.
iv. The tumbling beaker slipping off the edge of a table cannot be considered a point object because its size is not negligibly smaller than the height of the table.

36. Let us suppose that v_0 is the initial velocity of the particle, at time t = 0, and v be the velocity of the particle at t = t. The acceleration a of the particle can be written as,
$a = \frac{v - v_0}{t - 0}$
$\Rightarrow$ at = v - v_0
$\Rightarrow$ v = v_0 + at
This is the 1st kinematic equations for uniformly accelerated motion. Suppose, the particle travels a distance x in the time t.


The area of blue colored part gives the distance covered x.
So,
$x = v_0 t + \frac{1}{2}$ (v - v_0) $\times$ t = v_0t + $\frac{1}{2}$at $\times$ t
$\Rightarrow x = v_0 t + \frac{1}{2}at^2$

37. In first case. If u is initial velocity, then
v = $\frac{u}{2}$, s = 3 cm
As $v^2 - u^2 = 2as$
$\therefore (\frac{u}{2})^2 - u^2 = 2as$ or a = - $\frac{u^2}{8}$
In second case: v = 0, a = - $\frac{u^2}{8}$
Initial velocity = $\frac{u}{2}$
$0^2 - (\frac{u}{2})^2 = 2(-\frac{u^2}{8})$ s
or s = 1 cm
Thus the bullet will penetrate a further distance of 1 cm before coming to rest.

38. i. Displacement in first three seconds = Area of $\triangle$OAB
= $\frac{1}{2}(OB) \times (OA)$ = $\frac{1}{2}(3) \times (+30)$ = +45 m
ii. Acceleration = Slope of v - t graph
As, v - t graph is a straight line. So, consider the slope of line AB,
$\therefore$ Slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{0-30}{3}$ = -10 m/s^2
So, the acceleration is negative.

39. No, because the x-t graph does not represent the trajectory of the path followed by a particle. From the graph, it is noted that at t = 0, x = 0. The above graph can represent the motion of a body falling freely from a tower under gravity.

40. Given $x \propto t^2$
or $x = ct^2$,
where c is a constant
Velocity, v = $\frac{dx}{dt}$ = c $\times$ 2t
Acceleration, a = $\frac{dv}{dt}$ = 2c = a constant
Hence the object is moving with uniform acceleration.

41. Let a body falls freely from a tower of height h and takes n seconds to reach the ground. Then taking downward direction as the positive direction, we have
s = ut + $\frac{1}{2}gt^2$
Given u = 0 (free fall),
h = 0 + $\frac{1}{2}gn^2$ = $\frac{1}{2}gn^2$ ....(i)

The body travels half of its total path in the last second. Thus the body travels half of its total path in (n - 1) s.

$\frac{h}{2}$ = 0 + $\frac{g}{2}(n-1)^2 \Rightarrow$ h = g (n - 1)^2 ...(ii)

Equating (i) and (ii), we get,

$\Rightarrow$ $\frac{1}{2}gn^2$ = g(n - 1)^2

$\Rightarrow$ n = $\frac{\sqrt{2}}{\sqrt{2}-1} = 3.4142\ sec$

Hence the time of fall, n = 3.4142 sec

42. Here, $t = \sqrt{x} - 3$

$\sqrt{x} = t + 3$

x = (t + 3)^2

i. $v = \frac{dx}{dt} = 2(t+3)$

At t = 3 s, $v = 2(3+3) = 12 m/s$

ii. At t = 6 s, $v = 2(6+3) = 18 m/s$

43. As s = ut + $\frac{1}{2}$gt^2

$\therefore$ h = 0 $\times$ T + $\frac{1}{2}$gt^2

or T = $\sqrt{\frac{2h}{g}}$

Distance covered in time $\frac{T}{3}$,

$h' = 0 \times \frac{T}{3} + \frac{1}{2}g\left(\frac{T}{3}\right)^2$

= $\frac{gT^2}{18} = \frac{8}{18} \times \frac{2h}{g} = \frac{h}{9}$

Position of the ball after time $\frac{T}{3}$

= $h - \frac{h}{9} = \frac{8h}{9}$, above the ground.

44. Displacement of the particle in time (t)

S = area under $v - t$ graph

S = area of trapezium OABC

$S = \frac{1}{2}(v+u)t$

$S = \frac{1}{2}(u + at + u)t$ (v = u + at)

$S = \frac{1}{2}(2u + at)t$

$S = ut + \frac{1}{2}at^2$

45.

Speed	Velocity
1. It is the distance travelled by a body per unit time in any direction.	It is the distance travelled by a body per unit time in a fixed direction.
2. It is a scalar quantity.	It is a vector quantity.
3. Speed may be positive or zero but never negative.	Velocity may be positive, negative or zero.

46. v^2 - u^2 = 2as

We know a = $\frac{dv}{dt}$

Multiply and Divide by dx

$a = \frac{dv}{dt} \times \frac{dx}{dx}$

$a = \frac{dv}{dx} \times \vartheta$

adx = vdv

($\because$ $\frac{dv}{dt}$ = v)

Integrating with the limits

$a\int_{\lambda}^{x} dx = \int_{v}^{v} vdv$

a(x - x$_0$) = $\frac{v^2}{2} - \frac{v^2}{2}$

as $= \frac{v^2 - v^2}{2}$ ($\because$ (x - x$_0$) = s = displacement)

v^2 - u^2 = 2as

47. The given displacement is, $x = 18t + 15t^2$

Instantaneous velocity is the derivative of displacement with respect to time i.e.,

$\mathbf{v}_i = \frac{d\mathbf{x}}{dt}$ = 18 + 30t .....(i)

Instantaneous velocity at t = 0,

v = 18 + 30 × 0 = 18 m/sec

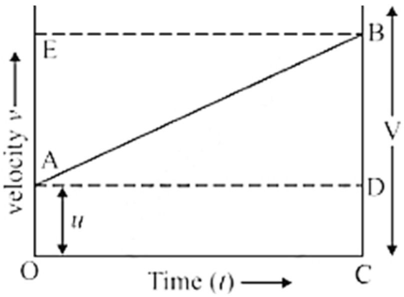
At t = 2 s, we have

v = 18 + 30×2 = 78 m/s

48. Let us divide the time interval of motion of an object under free fall into many equal intervals τ and find out the distances traversed during successive intervals of time. Since initial velocity is zero, we have $y = -\frac{1}{2}gt^2$

Using this equation, we can calculate the position of the object after different time intervals, 0 τ, 2τ 3τ. If we take (–1/2) $g\tau^2$ as y_0 - the position coordinate after the first-time interval τ, then the third column gives the positions in the unit of y_o. The fourth column gives the distances traversed in successive τs. We find that the distances are in the simple ratio 1: 3: 5: 7: 9: 11… as shown in the last column. This law was established by Galileo Galilei (1564-1642) who was the first to make quantitative studies of free fall.

49. We can see that the object P covers a distance of 10 m in every 15 min. In other words, it covers equal distance in equal intervals of time. So, the motion of object P is uniform. On the other hand, the object Q covers 7 m from 9:30 am to 9:45 am, 4 m from 9:45 am to 10:00 am and so on. In other words, it covers unequal distances in equal intervals of time. So, the motion of object Q is non-uniform.

50. 

displacement = area of ABCD

s = $\frac{1}{2}$(AB + CD)(AD)

= $\frac{1}{2}$(u + v)(t)

using v = u + at

s = $\frac{1}{2}$(u + u + at)(t)

= $\frac{1}{2}$(2u + at)(t)

= ut + $\frac{1}{2}at^2$

= $v^2 - \frac{u^2}{a}$

$v^2 = u^2 - 2as$

51. The given equation $x = at^2 - bt^3$

   i. If time t = 0, $x_0 = 0$

      If time t = 2 s, x_2 = 4a - 8b

      the displacement = $x_2 - x_0$ = 4a - 8b -0 = 4a - 8b

      Average speed in the given interval of time,(total displacement / total time)

      $v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{4a-8b}{2}$ = 2a - 4b

   ii. Instantaneous Speed is given by differentiation of x w.r.t time

      $v = \frac{dx}{dt} = \frac{d}{dt}\left(at^2 - bt^3\right)$ = $2at - 3bt^2$

      At t = 2s, v = 4a - 12b m/s.

52. Distance covered by car EC in first 5s is given by

s = ut + $\frac{1}{2}at^2$ = 0 + $\frac{1}{2}$ × 3 × t^2 = $\frac{3}{2}t^2$

At t = 5s, v = u + at = 0 + 3 × 5 = 15 ms^{-1}

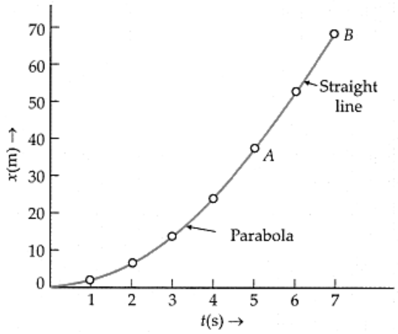
After t = 5 s, car covers a distance 15 m in each second upto t = 7 s.

Hence positions of the car at different instants of time will be

t(s)	0	1	2	3	4	5	6	7

x(m)	0	1.5	6.0	13.5	24.0	37.5	52.5	67.5

The figure shows the distance-time (x - t) graph for the car. For accelerated motion (0 to 5 s), the graph is a parabola (OA) and for uniform velocity (5 s to 7 s), the graph is straight line AB.



53. Given,

Velocity of the balloon = initial velocity of the food packet u = 14$\hat{j}$ ms^{-1}

Gravity acceleration a = -9.81$\hat{j}$ m/s^{-2}

Distance from ground h= -98 $\hat{j}$ m

Final velocity = v & time taken to reach the ground = t

Apply second kinematic equation

$v^2 - u^2 = 2ah$

$v = \sqrt{2ah + u^2}$

$v = \sqrt{2(-9.81)(-98) + 14^2}$

v = -46$\hat{j}$ m/s

Apply first kinematic equation

v = u + at

$t = \frac{v-u}{a} = \frac{-46-14}{-9.81}$ = 6.11

At velocity 46 m/s and in time 6.11 sec it will reach to the ground.

54. according to the question

$t = \sqrt{x} - 3$, therefore

$\sqrt{x} = t + 3$,

squaring both sides we get

$x = (t + 3)^2$

Now velocity of a particle is given by $v = \frac{dx}{dt} = \frac{d(t+3)^2}{dt} = 2(t+3)$

i. For t = 3 sec v = 2(3 + 3) = 12m/s
ii. For t = 6 sec , v = 2(6 + 3) = 18m/s

55. Here, distance traveled by the car = (2500 - 1700) km = 800 km

Time taken = 16 h

Average speed = $\frac{\text{Total distance covered}}{\text{Total time}}$

u = $\frac{800}{16}$ = 50 km/h

u = $\frac{50\times5}{1\times18}$ = 13.9 m/s

56. u = 72km/hr

= 20m/s

v = 0

s = 100m

$v^2 - u^2 = 2as$

$-20^2 = 200a$

-400 = 200a

a = -2m/s

57. Given: $t = ax^2 + bx$

$\therefore \quad \frac{dt}{dx}$ = 2 ax + b

Velocity, v = $\frac{dt}{dx}$ = $(2ax + b)^{-1}$
Acceleration = $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$ $\left[\because \frac{dx}{dt} = v\right]$
= v $\frac{dt}{dx}$ = $(2ax + b)^{-1}$
= $v(-1)(2ax + b)^{-2}.2a$
= $-2a(2ax + b)^{-3} = -2av^3$

58. Here it is given that, the position of an object is given by:

$x = 2t^2 + 3t$

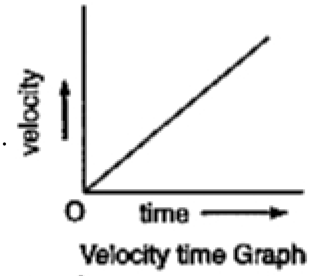
By differentiating x w.r.t t, we obtain
Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left(2t^2 + 3t\right)$
As velocity is time-dependent, it means that motion is non - uniform.

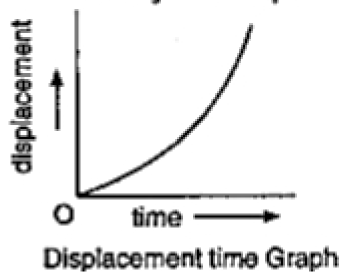
59. Any object can be considered as a point object if it covers large distances as compared to the size of it.
    1. The size of a carriage is very small as compared to the distance between two stations. Therefore, the carriage can be treated as a point sized object.
    2. The size of a monkey is very small as compared to the size of a circular track. Therefore, the monkey can be considered as a point sized object on the track.
    3. The size of a spinning cricket ball is comparable to the distance through which it turns sharply on hitting the ground. Hence, the cricket ball cannot be considered as a point object.

60. Yes, the object may be at rest relative to one object, and at the same time, it may be in motion relative to another object. For example, a passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to the objects outside the train. Rest and motion are relative terms.

Section C

61. Let the distance traveled by the vehicle before it stops be d_s. Then, using the equation of motion $v^2 = v_o^2 + 2\,ax$, and noting that v = 0, we have the stopping distance $d_s = \frac{-v_0^2}{2a}$
Thus, the stopping distance is proportional to the square of the initial velocity. Doubling the initial velocity increases the stopping distance by a factor of 4 (for the same deceleration). For the car of a particular make, the braking distance was found to be 10 m, 20 m, 34 m, and 50 m corresponding to velocities of 11, 15, 20, and 25 m/s which are nearly consistent with the above formula. Stopping distance is an important factor considered in setting speed limits, for example, in school zones.

62. 



63. Net distance crawled upwards in 2 min time
= 5 - 3 = 2cm
∴ Insect will crawl 2 × 10 = 20 cm upward in 2 × 10 = 20 min
In 21st min, insects cover 5 cm reach the crevice located at 25 cm.
∴ Total time taken = 20 + 1 = 21 min
The positions of insect at intervals of 1 min each will be

t (min)	x (cm)	t (min)	x (cm)
0	0	11	15
1	5	12	12

2	2	13	17
3	7	14	14
4	4	15	19
5	9	16	16
6	6	17	21
7	11	18	18
8	8	19	23
9	13	20	20
10	10	21	25

Position-time (x - t) graph can be drawn with the help of the above table.

64. a. The given x-t graph, shown in (a), does not represent one-dimensional motion of the particle. This is because a particle cannot have two positions at the same instant of time.
    b. The given v-t graph, shown in (b), does not represent one-dimensional motion of the particle. This is because a particle can never have two values of velocity at the same instant of time.
    c. The given v-t graph, shown in (c), does not represent one-dimensional motion of the particle. This is because speed being a scalar quantity cannot be negative.
    d. The given v-t graph, shown in (d), does not represent one-dimensional motion of the particle. This is because the total path length travelled by the particle cannot decrease with time.

65. ∴ Using the relation

$s_t = ut + \frac{1}{2}at^2$,

We get by putting t = n and n - 1,

$s_n = un + \frac{1}{2}an^2$...(i)

and $s_{n-1} = u(n - 1) + a(n - 1)^2$ ...(ii)

If S_n be the distance covered by the object in n^{th} second, then

$S_n = S_n - S_{n-1}$

$= un + \frac{1}{2}an^2 - [u(n - 1) + \frac{1}{2}a(n - 1)^2$

$= un + \frac{1}{2}an^2 - [un - u + \frac{1}{2}a(n^2 + 1 - 2n)]$

$= u - \frac{a}{2}(1 - 2n)$

or $s_{nth} = u + \frac{a}{2}(2n - 1)$ proved.

66. For simple harmonic motion (SHM) of a particle, acceleration (a) is given by the relation:

a = $-\omega^2 x$.....(i)

where ω angular frequency and $x =$ displacement

and velocity of the particle, v = $\frac{dx}{dt}$ .....(ii)

where $\frac{dx}{dt} =$ slope of x-t plot

Now at t = 0.3 s

In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. From equation (ii) again, velocity is the slope of x-t plot. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive.

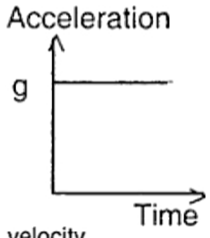
Now at t = 1.2 s

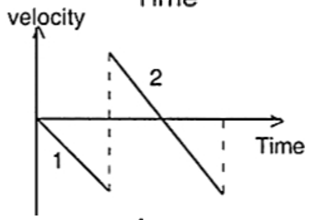
In this time interval, x is positive. Thus, the slope of the x-t plot i.e. the velocity of the particle will also be positive from equation (ii).

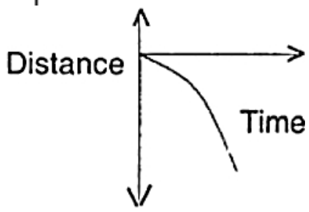
Therefore, both position and velocity are positive. However, using equation (i), acceleration of the particle comes to be negative.

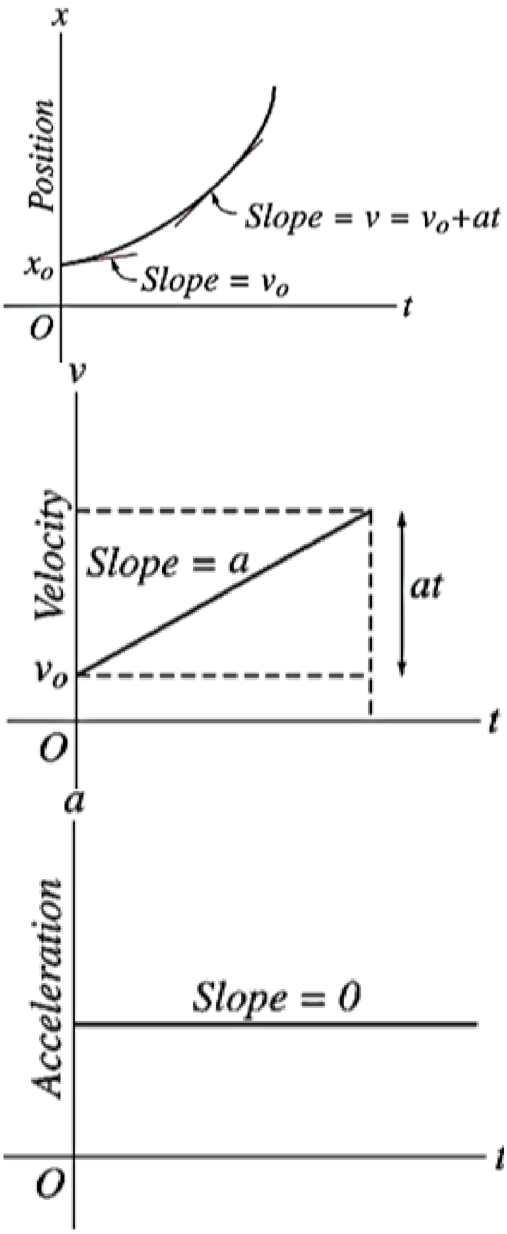
And at t = -1.2 s

In this time interval, x is negative and t is also negative. Hence, the slope of the x-t plot i.e. the velocity of the particle will be positive here from equation (ii). From equation (i), it can be inferred that the acceleration of the particle will be positive, as x is negative.

67. i. 

ii. 

iii. 

68. 

69. As $s_{nth} = u + \frac{a}{2}(2n - 1)$

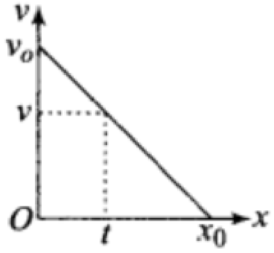
$\therefore x = u + \frac{a}{2}(2p - 1)$

$x = u + \frac{a}{2}(2p - 1)$ ...(i)

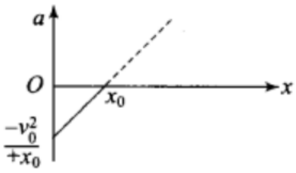
$y = u + \frac{a}{2}(2q - 1)$ ...(ii)

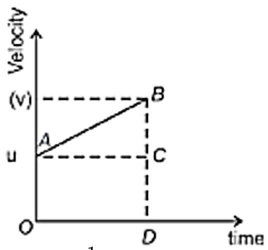
z = u + $\frac{a}{2}$(2r - 1) ...(iii)
Subtracting (iii) from (ii),
y - z = $\frac{a}{2}$(2q - 2r) or q - r = $\frac{y-z}{a}$
Subtracting (i) from (ii),
z - x = $\frac{a}{2}$(2r - 2p) or r - p = $\frac{z-x}{a}$
Subtracting (ii) from (ii),
x - y = $\frac{a}{2}$(2p - 2q) or q - p = $\frac{x-y}{a}$
Hence (q - r) x + (r - p)y + (p - q)z
= $\frac{(y-z)x}{a} + \frac{(z-x)y}{a} + \frac{(x-y)z}{a}$
= $\frac{(xy-xz)+(yz-xy)+(xz-yz)}{a}$= 0

70. In this question, we will use the equation of the straight line graph (linear equation).
y = mx + c.
In this equation, m is the slope of the graph and c is the interception on the y-axis.
Now according to the problem, initial velocity = v_0
Let the distance traveled in time t = x_0.
For the graph, $\tan\theta = \frac{v_0}{x_0} = \frac{v_0 - v}{x}$ ....(i)



Where, v is velocity and x is displacement at any instant of time t.
From Equation (i), we have
$v_0 - v = \frac{v_0}{x_0}x$
$\Rightarrow \quad v = \frac{-v_0}{x_0}x + v_0$
We know that,
Acceleration, $a = \frac{dv}{dt} = \frac{-v_0}{x_0}\frac{dx}{dt} + 0$
$\Rightarrow \quad a = \frac{-v_0}{x_0}(v)$
$= \frac{-v_0}{x_0}\left(\frac{-v_0}{x_0}x + v_0\right) = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$ ....(ii)
On comparing the equation (ii) with equation of a straight line y = mx + c we have
$m = \frac{v_0^2}{x_0^2} = +ve$, $c = -\frac{v_0^2}{x_0^2}$
Graph of 'a' versus x is given below.



71. 

Area = $\frac{1}{2}$(OA + BD) × AC
= $\frac{1}{2}$(u + v) × t
v = u + at...(i)

$= \frac{1}{2}(u + u + at) \times t$...using (i)

$S = ut + \frac{1}{2}at^2$

72. Given, helicopter is rising upwards steadily with a velocity, $u = 2\ ms^{-1}$.

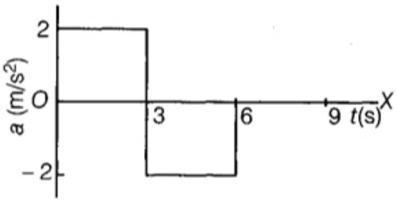
    i. When the food packet is released, its initial velocity is same as the velocity of helicopter i.e., $u = 2\ ms^{-1}$. Time taken by the packet to reach ground, t = 6 s. Let h_1 be the height of the helicopter at the time of releasing the food packet. Using the second equation of motion, $s = ut + \frac{1}{2}at^2$ and considering downward direction as positive, we have

    $h_1 = (-2) \times 6 + \frac{1}{2} \times 9.8 \times (6)^2 = -12 + 176.4 = 164.4$ m

    ii. During time, t = 6s, the helicopter has uniform motion and risen further with a distance $h' = ut = 2 \times 6 = 12$m. Hence, the height of the helicopter when food packet just reached the earth is,

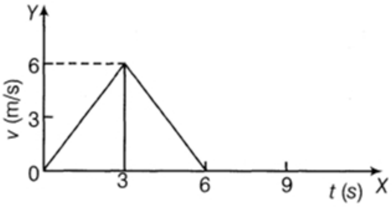
    $h_2 = h_1 + h' = 164.4 + 12 = 176.4$ m.

73. The acceleration -time graph is



The area enclosed between a-t curve gives change in velocity for the corresponding interval.

At t = 0, v = 0, hence final velocity at t = 3 s will increase to 6 m/s. In next 3 s, the velocity will decrease to zero. Thus, the velocity-time graph is



74. a. The given x-t graph shows that initially a body was at rest. Then, its velocity increases with time and attains an instantaneous constant value. The velocity then reduces to zero with an increase in time. Then, its velocity increases with time in the opposite direction and acquires a constant value. A similar physical situation arises when a football (initially kept at rest) is kicked and gets rebound from a rigid wall so that its speed gets reduced. Then, it passes from the player who has kicked it and ultimately gets stopped after sometime.

    b. In the given v-t graph, the sign of velocity changes and its magnitude decreases with a passage of time. A similar situation arises when a ball is dropped on the hard floor from a height. It strikes the floor with some velocity and upon rebound, its velocity decreases by a factor. This continues till the velocity of the ball eventually becomes zero.

    c. The given a-t graph reveals that initially the body is moving with a certain uniform velocity. Its acceleration increases for a short interval of time, which again drops to zero. This indicates that the body again starts moving with the same constant velocity. A similar physical situation arises when a hammer moving with a uniform velocity strikes a nail.

75. **<u>Given:</u>** Speed of woman = 5 km/h

Distance from home to office = 2.5 km

Time $= \frac{\text{Distance covered}}{\text{Speed of women}}$

Time = $\frac{2.5}{5}$ = 0.5 h = 30 min

When she returned, she covers the same distance in the evening by an auto.

Given Speed of auto = 25 km/h

Time $= \frac{\text{Distance Covered}}{\text{Speed of auto}}$

Time = $\frac{2.5}{25} = 0.1$ h = 6 min

The woman is at rest from 9:30 am to 5:00 pm and during this time, speed is zero

The x-t graph of the motion of women from home to office and office to home is shown below.