Solution

UNITS & MEASUREMENT COMBINED WORKSHEET

Class 11 - Physics

Section A

1. **(a)** atm L^2 per mol

Explanation:

Van der Wall's equation is

$\left(P+\frac{a}{V^2}\right)(V-b)=RT$

$\therefore$ [P] = $\left[\frac{a}{V^2}\right]$ or [a] = [P][V^2]

Unit of a = atm L^2 per mole.

2. **(a)** 4

Explanation:

In a number less than one (i.e. a decimal).

The zeroes on the left of non zero numbers are not significant figures, and zeroes to the right side of a nonzero number are significant figures.

In the number $0.0\underline{6900}$, two zeroes before six are not significant figure and two zero on the right side of 9 are significant figures. Significant figures are underlined.

3.

(b) [$M^{-1}L^3T^{-2}$]

Explanation:

$M^{-1}L^3T^{-2}$ is dimensional formula of universal gravitational constant .

4. **(a)** 5, 3, 2

Explanation:

There are three rules on determining how many significant figures are in a number:

- Non-zero digits are always significant.
- Any zeros between two significant digits are significant.
- A final zero or trailing zeros in the decimal portion ONLY are significant.

Keeping these rules in mind, we can say that only 5,3,2 are significant digits.

5.

(c) 129.6 new units

Explanation:

[Force] = [MLT^{-2}]

$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$

$= 1\left[\frac{1\text{ kg}}{1\text{ quintal}}\right]^1\left[\frac{1\text{ m}}{1\text{ km}}\right]^1\left[\frac{1\text{ s}}{1\text{ h}}\right]^{-2}$

$= \left[\frac{1}{100}\right]\left[\frac{1}{1000}\right]\left[\frac{1}{3600}\right]^{-2}$

$= \frac{3600\times3600}{100\times1000}$ = 129.6 new units

6.

(d) 4

Explanation:

There are three rules on determining how many significant figures are in a number:

- Non-zero digits are always significant.
- Any zeros between two significant digits are significant.
- A final zero or trailing zeros in the decimal portion ONLY are significant.

Keeping these rules in mind, we can say that there are 4 significant digits.

7.

(d) $x = -\frac{1}{2}, y = \frac{1}{2}$

Explanation:

Given: f = $am^x k^y$

Putting the dimensions of various quantities,

T^{-1} = 1.$[M]^x[MT^{-2}]$y [$\because$ k = $\frac{Force}{Distance}$]

$M^0T^{-1} = M^{x+y} + y^{-2y}$

x + y = 0 and -2y = -1

$\therefore y = \frac{1}{2}$ and $x = -\frac{1}{2}$

8.

(c) linear momentum

Explanation:

Same dimensions as linear momentum.

9.

(c) velocity $[LT^{-1}]$

Explanation:

Angles are dimensionless

[Bx] = 1 $\Rightarrow$ [B] = L^{-1}

[DT] = 1 $\Rightarrow$ [D] = T^{-1}

$\therefore \left[\frac{D}{B}\right]$ = $[LT^{-1}]$ $\Rightarrow$ Velocity

10.

(b) 1.7 g cm^{-3}

Explanation:

The answer to a multiplication or division is rounded off to the same number of significant figures as possessed by the least precise term used in the calculation. The final result should retain as many significant figures as are there in the original number with the least significant figures. In the given question, density should be reported to two significant figures

Density = $\frac{4.237\text{g}}{2.5\text{cm}^3}$ = 1.6948

After rounding off the number, we get density = 1.7

11.

(c) $[L^{-\frac{1}{2}}T^2]$

Explanation:

$L^{-\frac{1}{2}}T^2$

12.

(c) Pressure if a = 1, b = -1, c = -2

Explanation:

Pressure, $[P] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2}$

= $[M^1L^{-1}T^{-2}]$

$\therefore$ a = 1, b = -1, c = -2

13.

(c) $\frac{\sqrt{hG}}{c^{3/2}}$

Explanation:

Let $L = h^x G^y c^z$

$M^0 L^1 T^0 = [ML^2T^{-1}]^x [M^{-1}L^3T^{-2}]^y [LT^{-1}]^z$

$= M^{x-y} L^{2x+3y+z} T^{-x-2y-z}$

On solving, we get

$\therefore$ x - y = 0

2x + 3y + z = 1

- x - 2y - z = 0

$x = \frac{1}{2}; y = \frac{1}{2}; z = -\frac{3}{2}$

$\therefore L = \frac{\sqrt{hG}}{c^{3/2}}$

14.

(c) $\frac{r}{y}$

Explanation:

ωt and kx both are dimensionless. Out of the given options, only $\frac{r}{y}$ is dimensionless.

15.

(c) $[ML^5T^{-2}]$

Explanation:

$[a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2 = [ML^5T^{-2}]$

16.

(b) 4.6×10^{-5}

Explanation:

We will use the general rule of addition by making the powers same,

i.e., we will add 3.8×10^{-6} and 42×10^{-6}

$= 45.8 \times 10^{-6} = 4.58 \times 10^{-5}$

As there is only one number after the decimal in given figures we round off the answer to 4.6×10^{-5}.

17.

(b) $[MLT^{-1}]$

Explanation:

$E = mc^2 = hv$

$\therefore \left[\frac{hv}{c}\right] = [mc] = [MLT^{-1}]$

18.

(d) 4.4 cm

Explanation:

L + B = 2.331 + 2.1 = 4.431 = 4.4 cm

$\because$ B has 2 significant figures.

$\because$ L + B must have only 2 significant figures.

19.

(c) i and iv

Explanation:

Energy density

$= \frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2 T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}]$

Young's modulus,

$= \frac{F}{A} \cdot \frac{L}{l} = \frac{MLT^{-2}}{L^2} \cdot \frac{L}{L}$

= $[ML^{-1}T^{-2}]$

20.

(c) 1.9×10^{12}

Explanation:

According to the problem,

Young's modulus, Y = 1.9×10^{11} N/m^2

1N in SI system of units = 10^5 dyne in C.G.S system.

Hence, Y = $1.9 \times 10^{11} \times 10^5$ dyne/m^2

In C.G.S length is measured in unit cm, so we should also convert m into cm.

$\therefore$ Y = $1.9 \times 10^{11} \left(\frac{10^5 \text{dyne}}{10^4 \text{cm}^2}\right)$ [$\because$ 1m = 100 cm]

= 1.9×10^{12} dyne/cm^2

21. **(a)** Js

Explanation:

h = $\frac{\text{Energy}}{\text{Frequency}}$

Unit of h = $\frac{\text{J}}{\text{s}^{-1}}$ = Js

22. **(a)** $[ML^{-1}T^{-1}]$

Explanation:

$[\eta] = \frac{P(r^2 - x^2)}{4vl}$

$= \frac{[\text{ML}^{-1}\ \text{T}^{-2}][\text{L}^2]}{[\text{LT}^{-1}][\text{L}]}$

= $[ML^{-1}T^{-1}]$

23. **(a)** $[MLT^{-3}]$ and $[MLT^{-4}]$

Explanation:

[a] = $\frac{[F]}{[t]} = \frac{[\text{MLT}^{-2}]}{[\text{T}]}$ = $[MLT^{-3}]$

[b] = $\frac{[F]}{[t]^2} = \frac{[\text{MLT}^{-2}]}{[\text{T}]^2}$ = $[MLT^{-4}]$

24.

(b) Strain

Explanation:

Strain is a dimensionless quantity.

25.

(d) $(P^1 A^{1/2} T^{-1})$

Explanation:

According to the problem, fundamental quantities are momentum (p), area (A) and time (T) and we have to express energy in these fundamental quantities.

Let energy E,

$E \propto p^a A^A T^c \Rightarrow E = kp^a A^A T^c$

where, k is dimensionless constant of proportionality.

Dimensional formula of energy, [E] = $[ML^2T^{-2}]$ and [p] = $[MLT^{-1}]$

[A] = $[L^2]$, [T] = [T] and [E] = $[K][p]^a[A]^b[T]^c$

Putting all the dimensions, we get

$ML^2T^{-2} = [MLT^{-1}]^a[L^2]^b[T]^c$

$= M^aL^{a+2b}T^{a+c}$

According to the principle of homogeneity of dimensions, we get

a = 1 ......(i)

a + 2b = 2 .....(ii)

-a + c = -2 .....(iii)

By solving these equations (i), (ii) and (iii), we get

a = 1, b = $\frac{1}{2}$, c = -1

Dimesnional formula for E is $\left[p^1 A^{1/2} t^{-1}\right]$

26.

(d) $[ML^2T^{-1}]$

Explanation:

$[L] = [mvr] = [M][LT^{-1}][L]$

$= [ML^2T^{-1}]$

27.

(c) gravitational constant

Explanation:

Only G has dimensions of $[M^{-1}L^3T^2]$. The remaining three quantities are pure ratios.

28. **(a)** reference standard for the given physical quantity

Explanation: Unit is the reference used as the standard measurement of a physical quantity. The unit in which the fundamental quantities are measured are called fundamental unit and the units used to measure derived quantities are called derived units.

29.

(c) $[M^0L^0T^{-1}]$

Explanation:

$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{\mathrm{T}}$ = $[M^0L^0T^{-1}]$

30. **(a)** 57.3^o

Explanation:

We know that,

π radian = 180^o

1 radian = $\frac{180}{\pi} = \frac{180}{22} \times 7 = 57.3^o$

31.

(b) 34.2

Explanation:

The rules for rounding off are following.

- If the first non-significant digit is less than 5, then the least significant digit remains unchanged.
- If the first non-significant digit is greater than 5, the least significant digit is incremented by 1.
- If the first non-significant digit is 5, the least significant digit can either be incremented or left unchanged.
- All non-significant digits are removed.

So rounding off 34.216 upto 4 digits is 34.22 and upto 3 digits is 34.2

32.

(b) $[ML^2T^{-2}A^{-1}]$

Explanation:

$[\phi] = BA = \frac{F}{qv\sin\theta} A$

$= \frac{[\mathrm{MLT}^{-2}][\mathrm{L}^2]}{[\mathrm{AT}][\mathrm{LT}^{-1}]\cdot 1}$

$= [ML^2T^{-2}A^{-1}]$

33. **(a)** Force and impulse

Explanation:

[Force] = $[MLT^{-2}]$, [Impulse] = $[MLT^{-1}]$

34.

(c) $[FL^{-1}T^2]$

Explanation:

[F] = $[MLT^{-2}]$

$\therefore$ [M] = $[FL^{-1}T^2]$

35.

(d) torque and potential energy

Explanation:

[Torque] = [Potential energy] = $[ML^2T^{-2}]$

36.

(c) $[ML^2T^{-2}]$

Explanation:

[Torque] = $[ML^2T^{-2}]$

37.

(d) 5

Explanation:

There are three rules on determining how many significant figures are in a number:

- Non-zero digits are always significant.
- Any zeros between two significant digits are significant.
- A final zero or trailing zeros in the decimal portion ONLY are significant.

Keeping these rules in mind, we can say that there are 5 significant digits.

38.

(c) $[ML^2T^{-2}]$

Explanation:

$ML^{-2}T^{-2}$ is dimensional formula of torque.

39.

(b) quantities such as length, mass and time.

Explanation:

A physical quantity is a physical property of a phenomenon, body, or substance, that can be quantified by measurement. A physical quantity can be expressed as the combination of a magnitude expressed by a number – usually a real number – and a unit. All these given above can be expressed as explained so these are physical quantities.

40. **(a)** $[ML^2T^{-1}]$

Explanation:

$[h] = \frac{\text{Energy}}{\text{Frequency}} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

Section B

41. It is given that wavelength λ associated with a moving particle depends upon mass m, velocity v and Plank's constant (h).

Let, $\lambda = km^av^bh^c$ ....(1)

where k is a dimensional constant.

Writing dimensions of various terms, we get

$[M^0L^1T^0] = [M]^a [LT^{-1}]^b[ML^2T^{-1}]^c$

$[M^0L^1T^0] = M^{a+c}L^{b+2c}T^{-b-c}$

Using homogeneity rule and comparing dimensions of M,L and T we have

a + c = 0, b + 2c = 1, -b-c = 0

Solving, we get a = -1, b = -1, c = +1 putting these values in equation (1) we get

$\lambda = \frac{kh}{mv}$

42. Since, $e^{-(az/\theta)}$ is dimensionless (exponential function), we have $az/\theta = 1$

or, $a = \frac{\theta}{Z} = \frac{K}{L} = \left[L^{-1}K\right]$

We find that a/b = dimensions of pressure (because $e^{-(az/\theta)}$ is dimensionless)

$a/b = [ML^{-1}T^{-2}]$

Therefore,

$b = \frac{a}{\left[ML^{-1}T^{-2}\right]} = \frac{\left[L^{-1}K\right]}{\left[ML^{-1}T^{-2}\right]}$

$= [M^{-1}T^2K]$

43. Let F = $K\eta^a r^b v^c$, then

$M^1L^1T^{-2} = [ML^{-1}T^{-1}]^a[L]^b[LT^{-1}]^c$

$= M^a L^{-a + b + c} T^{-a - c}$

∴ a = 1, - a + b + c = 1, - a - c = -2

On solving, a = b = c = 1

Hence F = $K\eta rv = 6\pi\eta rv$ (Stoke's law)

44. i. Here, We have

$4.6 \times 0.128 = 0.5888 = 0.59$

The obtained result has been rounded off to have two significant digits (as in 4.6)

ii. Here, we have

$\frac{0.9995 \times 1.53}{1.592} = 0.96057 = 0.961$

The above result has been rounded off to three significant digits (as in 1.53).

iii. Here, we have

876 + 0.4382 = 876.4382 = 876

Since there is no decimal point in 876, therefore, the above result of addition has been rounded off to no decimal point.

45. Let $v = Kg^a R^b$

where K = a dimensionless constant.

Putting the dimensions,

$LT^{-1} = [LT^{-2}]^a [L]^b = L^{a + b} T^{-2a}$

Equating the powers of L and T,

a + b = 1, -2a = -1

∴ a = $\frac{1}{2}$, b = $\frac{1}{2}$

Hence, $v = K g^{1/2} R^{1/2}$

$v = K\sqrt{gR}$

46. Let x = $K\eta^a[E_k]^b$, then

$L^1 = [ML^{-1} T^{-2}]^a [ML^2 T^{-2}]^b$

$= M^{a + b} L^{-a + 2b} T^{-2a - 2b}$

∴ a + b = 0, - a + 2b = 1, -2a - 2b = 0

On solving, a = $-\frac{1}{3}$, b = $\frac{1}{3}$

Hence x = $K\eta^{-1/3}E_k^{1/3} = K\left[\frac{E_k}{\eta}\right]^{1/3}$

47. The principle of homogeneity states that the dimensions of each the terms of a dimensional equation on both sides are the same.

Using this principle the given equation will have same dimension on both sides.

On left side: h = [L], dimesion of legth

On right side: h_0 = [L], $v_0t = [LT^{-1}][T] = [L]$, $gT^2 = [LT^{-2}][T^2] = [L]$

Thus, the dimension on both sides quantities are same.

48. Given $\tau = I\alpha$

As torque, τ = Force $\times$ distance

$\therefore [\tau] = MLT^{-2} \cdot L = ML^2 T^{-2}$

Moment of inertia

I = Mass $\times$ distance^2

$\therefore$ [I] = ML^2

Angular acceleration,

$\alpha = \frac{\text{Angle}}{(\text{Time})^2}$

$\therefore [\alpha] = \frac{1}{T^2} = T^{-2}$

$[I\alpha] = ML^2T^{-2}$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

49. Dimensional formula in L.H.S. and R.H.S. by principal of homogeneity are equal.

$\therefore$ Dimension of y = dimensions of A sin (ωt - kx)

[L] = [L] $\times$ dimensions of (ωt - kx)

as (ωt - kx) are angle of sin (Trigonmetrical ratio)

So (ωt - kx) = No dimension or dimensions of ωt = dimensions of kx

$\frac{2\pi}{T}$ = Kx $\Rightarrow [M^0L^0T^0]$ = k[L]

Hence, Dimension of k = $\frac{[M^0L^0T^9]}{|L|}$ = $[M^0L^{-1}T^0]$, Dimension of ω = No dimension

50. It is given that the rotational kinetic energy, $E = \frac{1}{2}I\omega^2 \Rightarrow I = \frac{[E]}{[\omega^2]}$

Therefore, $I = \frac{[E]}{[\omega^2]} = \frac{[ML^2T^{-2}]}{[T^{-1}]^2}\left[\frac{ML^2T^{-2}}{T^{-2}}\right] = [ML^2]$

Its SI unit is Joule.

51. Let x = 2.5 $\times$ 10^{-6} = 0.0000025 (2 significant figures)

y = 4.0 $\times$ 10^{-4} = 0.00040 (2 significant figures)

$\therefore$ y - x = 0.00040 - 0.0000025 = 0.0003975

= 3.975 $\times$ 10^{-4} = 4.0 $\times$ 10^{-4} [Rounded off upto 2 significant figures]

52. Let $m = KF^aL^bT^c$

Substituting the dimension of, $[F] = [MLT^{-2}]$, $[L] = [L]\ and\ [T] = [T]$, we have

$[M] = [MLT^{-2}]^a[L]^b[T]^c$

$[M] = M^aL^{a+b}T^{-2a+c}$

On equating the powers on both sides, we get

a = 1, a + b = 0, -2a + c = 0

Solvign these equations, we get

a = 1, b = -1 and c = 2

Hence, dimensions of mass M are $[F^1L^{-1}T^2]$.

53. Let the period of oscillation T of a large fluid star depends on the radius of star, R, the mean density of fluid, ρ and universal gravitational constant, G as:

T = k R^a ρ^b G^c, where k is a dimensionless constant and a, b, c are their exponents.

Now, equating the dimensions on both the sides, we have

$[M^0\ L^0\ T^1] = [L]^a\ [M\ L^{-3}]^b\ [M^{-1}\ L^3\ T^{-2}]^c = M^{b-c}\ L^{a-3b+3c}\ T^{-2c}$

On comparing powers of M, L and T on both sides, we get,

b - c = 0 ...(i)

a - 3b + 3c = 0 ...(ii)

and - 2c = 1 ...(iii)

On simplifying these equations, we get c = $-\frac{1}{2}$, b = $-\frac{1}{2}$ and a = 0

Thus, period of oscillation, T = $k\rho^{-\frac{1}{2}}G^{-\frac{1}{2}} = \frac{k}{\sqrt{\rho G}}$

This is the required expression.

54. i. 4

Explanation: Significant figure- 2, 3, 7, 0. Trailing 0’s are significant. These 0’s increase the accuracy of the answer.

ii. 4

Explanation: Significant figure- 6, 3, 2, 0. Trailing 0’s are significant. These 0’s increase the accuracy of the answer.

55. i. Charge, q = Current $\times$ time

[q] = [AT]

ii. Potential,

V = $\frac{\text{Work}}{\text{Charge}}$

[V] = $\frac{ML^2T^{-2}}{AT}$ = $[ML^2A^{-1}T^{-3}]$

iii. Resistance,

R = $\frac{\text{Potential difference}}{\text{Current}}$

= $\frac{ML^2A^{-1}T^{-3}}{A}$ = $[ML^2A^{-2}T^{-3}]$,

iv. Capacitance,

C = $\frac{\text{Charge}}{\text{Potential}}$

[C] = $\frac{AT}{ML^2A^{-1}T^{-3}}$ = $[M^{-1}L^{-2}A^2T^4]$

56. Let $v_T = K(mg)^a\eta^b r^c$,

where K = a dimensionless constant.

Putting the dimensions of various quantities,

$LT^{-1} = [MLT^{-2}]^a\,[ML^{-1}T^{-1}]^b\,[L]^c$

or $M^0L^1T^{-1} = M^{a+b}L^{a-b+c}T^{-2a-b}$

Equating the powers of M, L and T on both sides, we get

a + b = 0, a - b + c = 1, - 2a - b = -1

On solving, a = 1, b = -1, c = -1

$\therefore$ $v_T = K(mg)^1\eta^{-1}r^{-1}$ or $v_T \propto \frac{mg}{\eta r}$

57. Let $T = Kh^a\,\rho^b\,g^c$

where a, b, c are the dimensions and k is dimensionless constant.

Writing the dimensions in (i) we get

$[M^0L^0T^1] = L^a(ML^{-3})^b(LT^{-2})^c$

$= M^bL^{a-3b+c}\,T^{-2c}$

Applying the principle of homogeneity of dimensions, we get,

b = 0, a − 3b + c = 0, −2c = 1, c = $\frac{1}{2}$

From a − 3b + c = 0, a − 3 $\times$ 0 - $\frac{1}{2}$ = 0, a = $\frac{1}{2}$1

Putting these value in (i) we get

T = $Kh^{1/2}\rho^0g^{-1/2}$

T = $K\left(\frac{\sqrt{h}}{\sqrt{g}}\right)$

58. Dimension of volume per second, V = $\frac{V}{T} = \frac{[L^3]}{[T]} = [L^3T^{-1}]$

Dimension of pressure P = $\frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

Dimension of radius , r = [L]

Dimension of coefficient of viscosity, η = $[ML^{-1}T^{-1}]$

Dimension of length of the pipe , l = [L]

$\therefore$ Dimension of R.H.S = $\frac{\pi Pr^4}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} = [M^0L^3T^{-1}]$, π and 8 are dimensionless entities.

And dimension of L.H.S., V = $[M^oL^3T^{-1}]$

Clearly, dimensions R.H.S = dimensions L.H.S, therefore the given equation is dimensionally accurate as it is perfectly in accordance with the principle of Homogeniety.

59. i. 6.2 g + 4.33 g + 17.456 g = 27.986 = 28.0 g

[rounded off to first decimal place]

ii. 187.2kg - 63.54kg = 123.66kg = 123.7kg
[rounded off to first decimal place]
iii. $75.5 \times 125.2 \times 0.51$ = 4820.826 = 4800
[rounded off upto two significant figures]
iv. $\frac{2.13 \times 24.78}{458.2}$ = 0.115193 = 0.115
[rounded off to three significant figures]

60. As n = $-D\frac{n_2-n_1}{x_2-x_1}$
$\therefore$ D = $\frac{n(x_2-x_1)}{(n_2-n_1)}$ [numerically]
Now n = number of particles per unit area per second
$\therefore$ [n] = $L^{-2}T^{-1}$
$n_2 - n_1$ = number of particles per unit volume
$\therefore$ [$n_2 - n_1$] = L^{-3}
$x_2 - x_1$ = position
$\therefore$ $x_2 - x_1$ = L
Hence, [D] = $\frac{L^{-2}T^{-1} \cdot L}{L^{-3}}$ = [L^2T^{-1}]

Section C

61. i. 18.4
ii. 143.4
iii. 19000
iv. 12.7
v. 248000
vi. 321.14
vii. 101.6×10^6
viii. 31.32×10^{-5}

62. it is given that, a planet moves around the sun in nearly circular orbitLet T = $Kr^a M^b G^c$
where K = a dimensionless constant
Dimensions of the various quantities are
[T] = T, [r] = L, [M] = M
[G] = $\frac{Fr^2}{m_1m_2} = \frac{MLT^{-2},L^2}{MM}$ = $M^{-1} L^3 T^{-2}$
Substituting these dimensions in equation (i), we get
[T] = $[L]^a [M]^b [M^{-1} L^3 T^{-2}]^c$
$M^0L^0T^1$ M^{b-c} L^{a+3c} T^{-2c}
Equating the dimensions of M, L and T, we get
b - c = 0, a + 3c = 0, -2c = 1
On solving,
a = $\frac{3}{2}$, b =$-\frac{1}{2}$, c = $-\frac{1}{2}$
$\therefore$ T = $Kr^{3/2} M^{-1/2} G^{-1/2}$
or T^2 = $\frac{K^2r^3}{MG} \Rightarrow T^2 \propto r^3$
As K = 2π, so T = $2\pi\sqrt{\frac{r^3}{GM}}$

63. i. Impulse - [MLT^{-1}]
ii. Power - [$M^1L^2T^{-3}$]
iii. Surface tension - [M^1T^{-2}]
iv. Coefficient of viscosity - [$M^1L^{-1}T^{-1}$]
v. Bulk modulus - [$M^1L^{-1}T^{-2}$]
vi. Force constant - [$M^1L^0T^{-2}$]

64. Let the velocity of the sound waves be given by

$v = Kd^aE^b$ ... (i)

where, K is a dimensionless constant.

Dimensions of the various quantities are given by

$[v] = LT^{-1}$, $[d] = ML^{-3}$

$[E] = \frac{\text{stress}}{\text{strain}} = \frac{\text{force}}{\text{area} \times \text{strain}} = \frac{MLT^{-2}}{L^2.1} = ML^{-1}T^{-2}$

Substituting these dimensions in equation (i), we get

$[LT^{-1}] = [ML^{-3}]^a [ML^{-1} T^{-2}]^b$

or $M^0L^1T^{-1} = M^{a+b} L^{-3a - b}T^{-2b}$

Equating the dimensions of M, L and T, we get

a + b = 0, -3a - b = 1, -2b = -1

On solving, a = $-\frac{1}{2}$, b = $\frac{1}{2}$

$\therefore$ v = $kd^{\frac{-1}{2}} E^{\frac{1}{2}}$

or v = $\sqrt{\frac{E}{d}}$.

65. During total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameters of both the sun and the moon must be equal.

$\therefore$ The angular diameter of the moon,

θ = Angular diameter of the sun

= 1920" = $1920 \times 4.85 \times 10^{-6}$ rad [$\because$ 1" = 4.85×10^{-6} rad]

Earth-moon distance, S = 3.8452×10^8 m

Diameter of the moon,

D = $\theta \times$ S

= $1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8$

= 3.581×10^6 m = 3581 km

66. Let the frequency of vibration of the string be given by

$\nu = Kl^am^bT^c$................(i)

where K is a dimensionless constant

Dimensions of the given quantities are

$\nu = [T^{-1}]$, l = [L], T = [T], m = $[ML^{-1}$ and tension(T) = $[MLT^{-2}]$

Substituting these dimensions in equation (i), we get

$[T^{-1}] = [L]^a[ML^{-1}]^b[MLT^{-2}]^c$

Expanding the above equation we get

$[M^0L^0T^{-1}] = [M^{b+c}L^{a-b+c}T^{-2c}]$

Equate dimensions of M, L and T on both sides of equation (homogeneity rule), we get

b + c = 0 ,a -b + c =0 and - 2c = -1

on solving, a = -1, $b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Put these values in equation (i) we get

$\nu = Kl^{-1}m^{-1/2}T^{1/2}$

or, $\nu = \frac{K}{l}\sqrt{\frac{T}{m}}$

67. Given that, the velocity v of water waves depends on the wavelength λ, density of water ρ and the acceleration due to gravity g.

Let v = $K\lambda^a\rho^bg^c$ ... (i)

where K = a dimensionless constant.

Dimensions of the various quantities are

$[v] = LT^{-1}$, $[\lambda] = L$, $[\rho] = ML^{-3}$, $[g] = LT^{-2}$

Substituting these dimensions in equation (i), we get

$[LT^{-1}] = [L]^a [ML^{-3}]^b[LT^{-2}]^c$

$M^0 L^1 T^{-1} = M^bL^{a - 3b + c} T^{-2c}$

Equating the powers of M, L and T on both sides, b = 0, a - 3b + c = 1, -2c = - 1

On solving, a = $\frac{1}{2}$, b = 0, c = $\frac{1}{2}$

$\therefore$ v = $K\lambda^{\frac{1}{2}}\rho^0 g^{\frac{1}{2}}$ = $K\sqrt{\lambda g}$

68. Let = $v = Kr^a\rho^b S^c$ ... (i)

where K = a dimensionless constant.

Dimensions of various quantities are

[v] = T^{-1}, [r] = L, [ρ] = ML^{-3}, [S] = MT^{-2}

Substituting these dimensions in equation (i), we get

$T^{-1} = [L]^a [ML^{-3}]^b [MT^{-2}]^c$

or $M^0L^0T^{-1} = M^{b+c} L^{a-3b} T^{-2c}$

By using principle of homogeneity of equations and Equating the powers of M, L and T on both sides,

b + c = 0, a - 3b = 0, - 2c = -1

On solving,

a = $-\frac{3}{2}$, b = $-\frac{1}{2}$, c = $\frac{1}{2}$, substituting these values in equation (i), we get

$\therefore v = Kr^{-3/2}\rho^{-1/2}S^{1/2} = K\sqrt{\frac{S}{\rho r^3}}$

69. a. The given equation is:

$y = a\sin\frac{2\pi t}{T}$

Dimension of y = $M^0L^1T^0$

Dimension of a = $M^0L^1T^0$

Dimension of sin $\frac{2\pi t}{T} = M^0L^0T^0$

$\because$ Dimension of L.H.S = Dimension of R.H.S

Hence, the given formula is dimensionally correct.

b. The given equation is:

$y = \left(\frac{a}{T}\right)\sin\left(\frac{t}{a}\right)$

Dimension of y = $M^0L^1T^0$

Dimension of $\frac{a}{T} = M^0L^1T^{-1}$

Dimension of $\frac{t}{a} = M^0L^{-1}T^l$

But the argument of the trigonometric function must be dimensionless, which is not so in the given case. Hence, the formula is dimensionally incorrect.

c. The given equation is:

$y = (a\sqrt{2})\left(\sin 2\pi\frac{t}{T} + \cos 2\pi\frac{t}{T}\right)$

Dimension of y = $M^0L^1T^0$

Dimension of a = $M^0L^1T^0$

Dimension of $\frac{t}{T} = M^0I^0T^0$

Since the argument of the trigonometric function must be dimensionless (which is true in the given case), the dimensions of y and a are the same. Hence, the given formula is dimensionally correct.

70. Radius of a hydrogen atom,

r = 0.5$\overset{\circ}{A}$ = 0.5 $\times$ 10^{-10} m

Volume of one atom = $\frac{4}{3}\pi r^3$

No. of atoms in 1 mole = 6.023 $\times$ 10^{23}

Volume of 1 mole of H-atoms = $N \times \frac{4}{3}\pi r^3$

= $6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times \left(0.5 \times 10^{-10}\right)^3$

= 3.154 $\times$ $10^{-7}m^3 \simeq 3 \times 10^{-7}$ m^3

71. Measurement is basically a comparison process. Without specifying a standard of comparison, it is not possible to get an exact idea about the magnitude of a dimensional quantity. For example, the mass of a human being is 40 kg, which is very small in comparison to the mass of the earth (6 $\times$ 10^{24} kg) but very large in comparison to the mass of an electron (9.1 $\times$ 10^{-31} kg).

i. An atom is a very small object when we compare it with a cricket ball.

ii. A jet plane moves with a speed greater when we compare it with a motor bike.
iii. The mass of Jupiter is very large as compared to the mass of Venus.
iv. The air inside this room contains a large number of molecules than in one mole of air.
v. The given statement is already correct.
vi. Speed of sound is less than the speed of light. Speed of sound 346 m/ sec while speed of light is 3 x 10^8 m / sec in air at 25 ^{0}C, hence statement is true.

72. According to question, the mass M of the largest stone that can be moved by a flowing river depends upon v the velocity, ρ the density of water and on g, the acceleration due to gravity

Let M = $Kv^a \rho^b g^c$

where K = a dimensionless constant.

Dimensions of the various quantities are

[M] = M, [v]= LT^{-1} [ρ] = ML^{-3}, [g] = LT^{-2}

Substituting these dimensions in equation (i), we get

[M] = $[LT^{-1}]^a[ML^{-3}]^b [LT^{-2}]^c$

$M^1 L^0 T^0 = M^b L^{a - 3b + c} T^{-a -2 c}$

Equating the powers of M, L and T, we get

b = 1, a - 3b + c = 0, - a - 2c = 0

On solving, a = 6, b = 1, c = -3

$\therefore$ M = $Kv^6\rho^1 g^{-3}$

Hence M $\propto$ v^6

73. i. Given, $\lambda = \frac{h}{mv}$

LHS as wavelength is a distance λ = [L]

Also RHS, $\frac{h}{mv} = \frac{\text{Planck's constant}}{\text{Mass} \times \text{Velocity}}$

$= \frac{[ML^2 T^{-1}]}{[M]\times[LT^{-1}]}$ = [L]

$\therefore$ LHS = RHS

Hence, the given equation is dimensionally correct.

ii. Here, v = $\sqrt{\frac{2GM}{R}}$

LHS v = [LT^{-1}] RHS = $\left[\frac{2GM}{R}\right]^{1/2}$

G = $\left[M^{-1} L^3 T^{-2}\right], R = [L], M = [M]$

$= \left[\frac{M^{-1} L^3 T^{-2} M}{L}\right]^{1/2}$ = $[L^2T^{-2}]^{1/2}$ = [LT^{-1}]

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence, the equation is dimensionally correct.

74. given that Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe therefore

Let N_R = $K\rho^a v^b \eta^c D$ ... (i)

where K = a dimensionless constant.

Dimensions of various quantities are

[N_R] = 1 = $M^0 L^0 T^0$

[ρ] = ML^{-3}, [v] = LT^{-1}

[η] = $ML^{-1} T^{-1}$, [D] = L

Substituting these dimensions in equation (i), we get

$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [ML^{-1} T^{-1}]^c [L]$

$[M^0 L^0 T^0] = M^{a + c} L^{-3 a+ b - c + 1} T^{-b - c}$

Equating the powers of M, L and T, we get

a + c = 0, -3a + b - c + 1 = 0, -b - c = 0

On solving, a = 1, b= 1, c = - 1

$\therefore$ $N_R = K\rho^1 v^1 \eta^{-1}$ D = $K\frac{\rho v D}{\eta}$

or $N_R \propto \frac{\rho v D}{\eta}$

75. Let h = $K\rho^a S^b g^c \times \frac{1}{r}$ ... (i)

where K = a dimensionless constant.

Dimensions of the various quantities are

[h] = L, [ρ] = ML^{-3}, [S] = MT^{-2}, [g] = LT^{-2}, [r] = L

Substituting the dimensions of various quantities in (i), we get

L = $[ML^{-3}]^a [MT^{-2}]^b [LT^{-2}]^c [L^{-1}]$

or $M^0L^1T^0 = M^{a + b} L^{-3a + c + 1} T^{-2b - 2c}$

Equating the powers of M, L and T, we get

a + b = 0, - 3a + c - 1 = 1, - 2b - 2c = 0

On solving, a = -1, b = 1, c = -1

$\therefore$ h = $K\rho^{-1}S^1g^{-1} \times \frac{1}{r}$ or h = $K\frac{S}{r\rho g}$