Solution

VECTOR ALGEBRA

Class 11 - Physics

Section A

1.

(c) current

Explanation:

Electric current is a scalar quantity. It represents the direction of flow of positive charge but it is treated as a scalar quantity because current follows the laws of scalar addition and not the laws of vector addition, because the angle between the wires carrying current does not affect the total current in the circuit.

2.

(c) Impulse

Explanation:

Since force is a vector quantity, the impulse is also a vector in the same direction. Impulse applied to an object produces an equivalent vector change in its linear momentum, also in the same direction.

3.

(b) polar vector

Explanation:

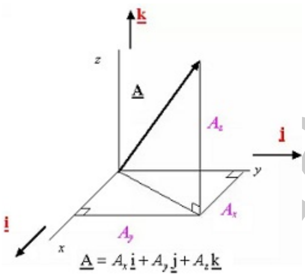
Acceleration due to gravity is a polar vector.

4.

(b) $\sqrt{A_x^2 + A_y^2 + A_z^2}$

Explanation:

<u>Rectangular components of a Vector:</u> The projections of vector a along the x, y, and z directions are A_x, A_y, and A_z, respectively.



The magnitude of vector = $\sqrt{A_x^2 + A_y^2 + A_z^2}$

5.

(b) axial vector

Explanation:

As $\vec{L} = \vec{r} \times \vec{p}$

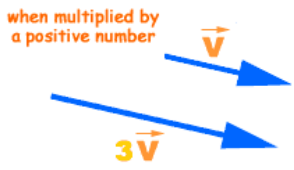
Hence, angular momentum is an axial vector.

6.

(d) gives a vector $\overrightarrow{v'}$ = $\lambda\vec{v}$ in the same direction as $\vec{v}$

Explanation:

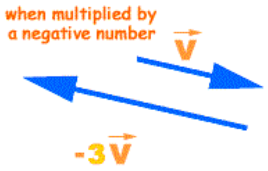
When a vector is multiplied by a positive number (for example 2, 3,5, 60 unit etc.) or a scalar only its magnitude is changed but its direction remains the same as that of the original vector.



7. **(a)** gives a vector $\overrightarrow{v'} = \lambda\vec{v}$ in a direction opposite to $\vec{v}$

Explanation:

If a vector is multiplied by a negative number (for example -2, -3, -5, -60 unit etc.) or a scalar not only its magnitude is changed but its direction also reversed.



8.

(c) $\vec{A} = A_x\hat{i} + A_y\hat{j}$

Explanation:

In a two-dimensional coordinate system, any vector can be broken into x -component and y -component.

Vector $\vec{A}$ can be represented as

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

Where A_x is component in direction of x-axis and $\hat{i}$ is a unit vector in same direction of x-axis and A_y is component in direction of y-axis and $\hat{j}$ is a unit vector in same direction of y-axis.

9. **(a)** any chosen direction

Explanation:

A unit vector is a vector that has a magnitude of 1 unit. A unit vector is also known as a direction vector. It is represented using a lowercase letter with a cap ('^') symbol along with it. A vector can be represented in space using unit vectors. Any vector can become a unit vector by dividing it by the vector's magnitude as follows:

$\text{Unit vector} = \frac{\text{vector}}{\text{magnitude of the vector}}$

$\hat{i}$ = a unit vector directed along the positive x-axis

$\hat{j}$ = a unit vector directed along the positive y-axis

$\hat{k}$ = a unit vector directed along the positive z-axis

10. **(a)** $\hat{i} = \hat{j} = \hat{k}$ = 1

Explanation:

All the three unit vectors have the magnitude as unity, $\hat{i} = \hat{j} = \hat{k}$ = 1

11.

(b) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

Explanation:

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$= -18\hat{i} - 13\hat{j} + 2\hat{k}$

12.

(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction

Explanation:

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = -|\vec{A}||\vec{B}|$

$\therefore \quad \cos\theta = -1$

or $\theta = 180^\circ$

13.

(b) 60°

Explanation:

Given: $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

$\therefore AB \sin\theta = \sqrt{3} AB \cos\theta$

or $\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

14.

(c) 75°52'

Explanation:

As $R^2 = P^2 + Q^2 + 2PQ \cos\theta$

$\therefore 24^2 = 12^2 + 18^2 + 2 \times 12 \times 18 \cos\theta$

or 576 - (144 + 324) = 432 $\cos\theta$

or $\cos\theta = \frac{108}{432}$ = 0.25

$\therefore \theta = \cos^{-1}(0.25) = 75^o 52'$

15.

(d) 120^o

Explanation:

As $\vec{R} = \vec{P} + \vec{Q}$

and $|\vec{P}| = |\vec{Q}| = |\vec{R}|$,

$R^2 = P^2 + Q^2 + 2\ PQ\cos\theta$

or $P^2 = P^2 + P^2 + 2P^2 \cos\theta$

or $\cos\theta = -\frac{1}{2}$

$\therefore \theta = 120^\circ$

16. **(a)** 90°

Explanation:

$\vec{A} \cdot \vec{B} = 4 \times 3 + 4 \times 1 - 4 \times 4 = 0$

$\Rightarrow \vec{A} \perp \vec{B}$ or $\theta = 90^o$

17.

(d) 45^o

Explanation:

Given: $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$

$\therefore$ AB $\sin\theta$ = AB $\cos\theta$ or $\tan\theta = 1$

$\therefore \theta = 45^\circ$

18.

(c) western direction

Explanation:

By right hand rule, the direction of $\vec{A} \times \vec{B}$ is towards west.

19.

(c) 90°

Explanation:

$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{A} \cdot \vec{B} = 3 \times 3 + 4 \times 4 + 5 \times (-5) = 0$

$\Rightarrow \vec{A} \perp \vec{B}$

20.

(c) a scalar

Explanation:

Scalar product means dot product and the dot product of 2 vectors gives a scalar. For example dot product of force and displacement gives work which is scalar.

21.

(d) zero

Explanation:

As scalar triple product is cyclic,

$(\vec{B} \times \vec{A}) \cdot \vec{A} = (\vec{A} \times \vec{B}) \cdot \vec{A} = (\vec{A} \times \vec{A}) \cdot \vec{B}$

$= \vec{0} \cdot \vec{B} = 0$

22.

(d) $\hat{j} \times \hat{k} = \hat{i}$

Explanation:

In a clockwise system,

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

And $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Therefore, the right option is $\hat{j} \times \hat{k} = \hat{i}$

23.

(d) commutative and distributive

Explanation:

The scalar product is:

a. Commutative:
   $\vec{A}.\vec{B} = \vec{B}.\vec{A}$
   |A||B| cos θ = |B||A| cos θ
b. Distributive
   $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

24. **(a)** $|\mathbf{A}|\,|\mathbf{B}|$ cosθ

Explanation:

The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it times the magnitude of the other vector. This can be expressed in the form $\vec{A}.\vec{B} = |A||B|\cos\theta$

25.

(d) cos^{-1} $(\frac{5}{12})$

Explanation:

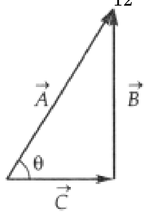
$13^2 = 12^2 + 5^2$

$\Rightarrow |\vec{A}|^2 = |\vec{B}|^2 + |\vec{C}|^2$

$\Rightarrow \vec{B} \perp \vec{C}$

$\therefore \cos\theta = \frac{|\vec{B}|}{|\vec{C}|} = \frac{5}{12}$

$\theta = \cos^{-1}\frac{5}{12}$



26.

(c) 90°

Explanation:

Given: $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow$ $a^2 + b^2 + 2ab \cos\theta = a^2 + b^2$ ab $\cos\theta$

$\Rightarrow$ 4ab $\cos\theta = 0$

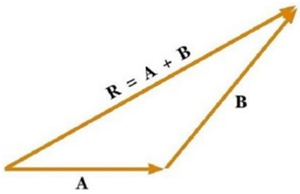
$\therefore \theta = 90^\circ$

27.

(c) tail is at the head of the vector $\vec{A}$

Explanation:

Triangle law of vector addition states that when two vectors are represented as two sides of the triangle with the order of magnitude and direction, then the third side of the triangle represents the magnitude and direction of the resultant vector. Thus resultant vector is R= a+ b



Taken in same order mean tail of Vector B should be placed at the head of vector A as shown in image.

28. **(a)** $(12\hat{i} + 16\hat{j} - 3\hat{k})$

Explanation:

Given $\vec{A} + 3\vec{B} - \vec{C} = 0$

$\therefore \vec{C} = \vec{A} + 3\vec{B} = (3\hat{i} + \hat{j} + 3\hat{k}) + 3(3\hat{i} + 5\hat{j} - 2\hat{k})$

$= 12\hat{i} + 16\hat{j} - 3\hat{k}$

29.

(c) are equal to each other

Explanation:

Given: $\left(\vec{F}_1 + \vec{F}_2\right) \perp \left(\vec{F}_1 - \vec{F}_2\right)$

$\therefore \left(\vec{F}_1 + \vec{F}_2\right) \cdot \left(\vec{F}_1 - \vec{F}_2\right) = 0$

or $F_1^2 - \vec{F}_1 \cdot \vec{F}_2 + \vec{F}_2 \cdot \vec{F}_1 - F_2^2 = 0$

or $F_1^2 = F_2^2$

$\Rightarrow F_1 = F_2$

30. **(a)** B = 0

Explanation:

We have to identity statements which are always true. It is given that $|\vec{A} + \vec{B}| = |\vec{A}|$, it could be true in two conditions that is either $\vec{B} = 0$ or $\vec{B} = -2\vec{A}$.

For forming a single condition we will multiply them, as either one of them is true it will uphold the necessary condition

We know $\vec{B} = 0, \vec{B} - 2\vec{A} = 0$ (from previous equations)

Therefore their magnitude's product will also be zero.

$|\vec{B}|(|\vec{B}| - 2|\vec{A}|) = 0$ (This will always be true)

$|\vec{B}|^2 - 2|\vec{A}||\vec{B}|$ = 0

Therefore,

$|\vec{A}||\vec{B}| \leq 0$ (Equality is true for B = 0)

Above condition is always true

31. **(a)** -1

Explanation:

Vector subtraction is defined in the following way.

- The difference of two vectors, A - B , is a vector C that is, C = A - B
- The addition of two vector such that C = A + (-B). B has been taken in opposite direction.

Thus vector subtraction can be represented as a vector addition.

32.

(b) 100 m

Explanation:

When the cyclist takes the seventh turn, he is at a 100 m distance from the initial position.

33.

(b) change in position

Explanation:

A displacement vector is a change in the position vector.

34.

(d) 8.54 m s^{-1}, 70° with x-axis

Explanation:

Position vector is given by, $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 4.0\hat{k}$

We know velocity is given by:-

$\vec{v} = \frac{dr}{dt}$

So, $\vec{v} = 3.0\hat{i} + 4t\hat{j}$

Velocity after 2 seconds is:

$\vec{v} = 3\hat{i} + 8\hat{j}$

Magnitude of velocity = $\sqrt{(3)^2 + (8)^2}$ = $\sqrt{73}$= 8.54 ms^{-1}

Direction is given by

$\theta = tan^{-1}(\frac{8}{3}) = 69.5 \approx 70^\circ$ with x-axis

35.

(b) $\frac{11}{5}(\hat{i} + \hat{j})$

Explanation:

$\vec{v}_{ao} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$

$= \frac{(13\hat{i}+14\hat{j})-(2\hat{i}+3\hat{j})}{5-0} = \frac{11}{5}(\hat{i} + \hat{j})$

36.

(d) is either less or equal to the path length of the particle between two points.

Explanation:

The maximum possible value for displacement is the distance travelled, so it cannot be of a greater value than distance (path length). The magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled.

37.

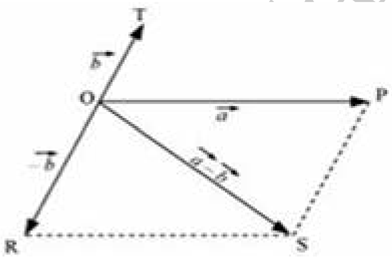
(d) $(10\hat{i} - 6\hat{j})$

Explanation:
Bird's displacement is
$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
$= (7\hat{i} - 2\hat{j} - 3\hat{k}) - (-3\hat{i} + 4\hat{j} - 3\hat{k})$
$= 10\hat{i} - 6\hat{j}$

38. No. A scalar quantity does not depend on the frame of reference.
39. No. This is because the addition of 1 rotations about different axes does not obey commutative law.
40. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$ = 0 + 0 = 0
because $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular to each other.
41. L.H.S. $= (\vec{A} \times \vec{B})^2$
$= |\vec{A} \times \vec{B}|^2 = (AB\sin\theta)^2$
$= A^2B^2\left(1 - \cos^2\theta\right)$
$= A^2B^2 - (AB\cos\theta)^2$
$= A^2B^2 - (\vec{A} \cdot \vec{B})^2$ = R.H.S.
42. May or may not be. The dot product of two vectors gives a scalar quantity while the cross product gives a vector quantity.
43. When they are mutually perpendicular to each other.

Section B

44. Vector quantities are those quantities which are characterised by magnitude as well as direction and which obey the vector law of addition e.g., displacement, velocity, acceleration, force, momentum, impulse, angular momentum, torque, electric field intensity, dipole moment are all vectors.
45. We know that time always flows on and on i.e. from past to present and then to future.
Therefore, a direction can be assigned to time. Since, the direction of time is unique and it is unspecified or unstated. That is why, time cannot be a vector though it has a direction.
46. $|\vec{a} + \vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2$
$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$
$= -2|\vec{a}||\vec{b}|(1 - \cos\theta)$
$= -2|\vec{a}||\vec{b}|\sin^2\frac{\theta}{2}$ = a negative quantity
Hence $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
47. $|\Delta\bar{a}| = ad\theta$
and
$\Delta a = 0$
48. Let two vectors $\bar{a}$ and $\bar{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



The following relations can be written for the given parallelogram.
OS + PS > OP ....(i)
OS > OP - Ps ....(ii)
$|\bar{a} - \bar{b}| > |\bar{a}| - |\bar{b}|$ ....(iii)
The quantity on the LHS is always positive and that on the RHS can be positive or negative. To make both quantities positive, we take modulus on both sides as:
$|\bar{a} - \bar{b}|| > ||\bar{a}|| - ||\bar{b}||$
$|\bar{a} - \bar{b}| > ||\bar{a}|| - |\bar{b}||$ ....(iv)
If the two vectors act in a straight line but in the opposite directions, then we can write:
$|\bar{a} - \bar{b}| = ||\bar{a}|| - ||\bar{b}||$ ...(v)
Combining equations (iv) and (v), we get:
$|\bar{a} - \bar{b}| \geq ||\bar{a}|| - ||\bar{b}||$

49. i. If they directed opposite to each other at an angle 180°, resultant force is zero

ii. When the two forces act in the same direction, the magnitude of their resultant

= Magnitude of $2\vec{F}$

iii. For the resultant to be F, from parallelogram law of vector addition

$F^2 = F^2 + F^2 + 2F^2\cos\theta$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = 120^\circ$

They should be inclined at an angle 120° with respect to each other.

50. Walking of a man is an example of resolution of forces. A man while walking presses the ground with his feet backward by a force F at an angle θ with ground, in action.

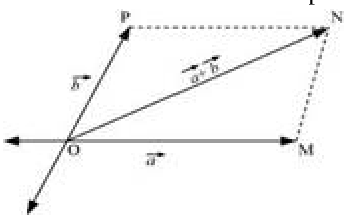
The ground in reaction exerts an equal and opposite force R (= F) on the feet.

Its horizontal component H = Rcos θ enables the person to move forward while the vertical component V = Rsin θ balances his weight because R is resolved into two rectangular components.

Application of resolution of vector

It is easier to pull a lawn mower than to push.

51. Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we can write:

$|\overrightarrow{\mathrm{OM}}| = |\vec{\mathrm{a}}|$ ....(i)

$|\overrightarrow{\mathrm{MN}}| = |\overrightarrow{\mathrm{OP}}| = |\overrightarrow{\mathrm{b}}|$ ....(ii)

$|\overrightarrow{\mathrm{ON}}| = |\vec{a} + \vec{\mathrm{b}}|$ ....(iii)

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

OM + MN > ON

$|\overrightarrow{ON}| < |\overrightarrow{OM}| + |\overrightarrow{MN}|$

$|\vec{\mathrm{a}} + \vec{\mathrm{b}}| < |\vec{\mathrm{a}}| + |\vec{\mathrm{b}}|$ ....(iv)

If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write:

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ ....(v)

Combining equations (iv) and (v), we get:

$|\vec{\mathrm{a}} + \vec{\mathrm{b}}| \leq |\vec{\mathrm{a}}| + |\vec{\mathrm{b}}|$

This equality applies, if a and b are acting in the same direction e.g. angle between them = 0.

52. A unit vector is a vector having unit magnitude. It is used to denote the direction of a given vector.

The unit vector of a given vector $\vec{A}$ is obtained on dividing the given vector by its magnitude. Thus,

The unit vector of $\vec{A}$ is, $\hat{A}$ = $\frac{\vec{A}}{|\vec{A}|}$.

53. $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \mathbf{2}\hat{\mathbf{i}} + \mathbf{6}\hat{\mathbf{j}} + \hat{\mathbf{k}}$ ...(i)

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 11\hat{\mathbf{k}}$ ...(ii)

Adding equations (i) and (ii)

$\mathbf{2}\vec{\mathbf{A}} = 6\hat{\mathbf{i}} + \mathbf{8}\hat{\mathbf{j}} - \mathbf{10}\hat{\mathbf{k}}$

$\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Subtracting (ii) from (i)

$\mathbf{2}\vec{B} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \mathbf{12}\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{B}} = -\hat{\mathbf{i}} + \mathbf{2}\hat{\mathbf{j}} + \mathbf{6}\hat{\mathbf{k}}$

Therefore, $|\vec{\mathbf{A}}| = \mathbf{5}\sqrt{\mathbf{2}}$

$|\vec{\mathbf{B}}| = \sqrt{\mathbf{41}}$

$\vec{A}.\vec{B}$ = -3 + 8 - 30

= -25

54. Given

$\overrightarrow{\mathbf{A}} = \mathbf{3\hat{i}} + \mathbf{2\hat{j}}$ and $\overrightarrow{\mathbf{B}} = -3\hat{\mathbf{i}} + \mathbf{7j}$

We know the area of parallelogram formed by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is given by $|\vec{A} \times \vec{B}|$

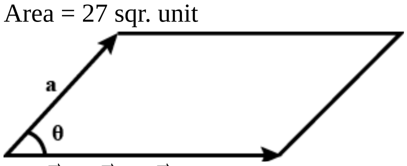
$\vec{A} \times \vec{B} = (3\hat{\mathbf{i}} \times \mathbf{2\hat{j}}) \times (-\mathbf{3\hat{i}} + \mathbf{7\hat{j}})$

$= \mathbf{21\hat{k}} + \mathbf{6\hat{k}}$

$|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}$ = 27

$\Rightarrow$ area of parallelogram = 27

Area = 27 sqr. unit



55. As $\vec{P} + \vec{Q} = \vec{R}$

$\therefore \vec{R} - \vec{Q} = \vec{P}$

and $(\vec{R} - \vec{Q}) \cdot (\vec{R} - \vec{Q}) = \vec{P} \cdot \vec{P}$

or $\vec{R} \cdot \vec{R} - \vec{R} \cdot \vec{Q} - \vec{Q} \cdot \vec{R} + \vec{Q} \cdot \vec{Q} = \vec{P} \cdot \vec{P}$

or $R^2 - 2\vec{R} \cdot \vec{Q} + Q^2 = P^2$

or $\cos\theta = \frac{R^2+Q^2-P^2}{2RQ}$

$= \frac{13^2+12^2-5^2}{2\times13\times12} = \frac{288}{2\times13\times12} = \frac{12}{13}$

$\therefore \theta = \cos^{-1}\frac{12}{13}$

56. $\vec{A} = 4\hat{i} + 3\hat{j} + \hat{k},\ A = \sqrt{4^2 + 3^2 + 1^2} = \sqrt{26}$

$\vec{B} = 12\hat{i} + 9\hat{j} + 3\hat{k},\ B = \sqrt{12^2 + 9^2 + 3^2} = \sqrt{234}$

$\vec{A} \cdot \vec{B}$ = 48 + 27 + 3 = 78

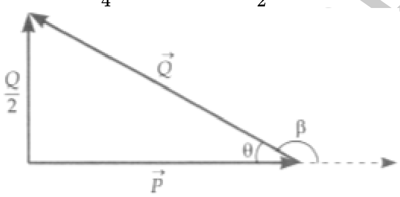
$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{78}{\sqrt{26\sqrt{234}}} = \frac{78}{78} = 1$

So, $\theta = 0^0$

So vectors are parallel to each other.

57. Clearly, $P^2 + \left(\frac{Q}{2}\right)^2 = Q^2$

or $P^2 = \frac{3}{4}Q^2$ or $P = \frac{\sqrt{3}}{2}Q$



$\therefore \tan\theta = \frac{Q/2}{P} = \frac{Q/2}{\sqrt{3}Q/2} = \frac{1}{\sqrt{3}}$

or $\theta = 30^\circ$

Angle between $\vec{P}$ and $\vec{Q}$,

$\beta = 180^\circ - \theta = 180^\circ - 30^\circ$ = 150^o

58. Given that,

Force $\vec{\mathrm{F}} = 5\hat{\mathrm{i}} + 4\hat{\mathrm{j}}$

Displacement $\vec{\mathrm{s}} = 3\hat{\mathrm{i}} + 4\hat{\mathrm{k}}$

Time t = 3s

Now, the work done is

W = $\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{s}}$

W = $(5\hat{i} + 4\hat{j} + 0\hat{k}) \cdot (3\hat{i} + 0\hat{j} + 4\hat{k})$

W = 15 + 0 + 0

W = 15J

Now, the power is

$P = \frac{W}{t}$

$P = \frac{15}{3}$

P = 5 watt

Hence, the power is 5 watt.

59. We know that, $\vec{A} \cdot \vec{B}$ = AB cos θ, when vectors are orthoganal, θ = 90^o.

So, $\vec{A} \cdot \vec{B}$ = AB cos 90^o = 0, when vectors are parallel, then, θ = 0^o.

So, $\vec{A} \cdot \vec{B}$ = AB cos 0^o = AB (maximum)

60. As we know that, $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\,cos\,\theta}$

and $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\,cos\,\theta}$

But as per question, we have

$\sqrt{A^2 + B^2 + 2AB\,cos\,\theta} = \sqrt{A^2 + B^2 - 2AB\,cos\,\theta}$

Squaring both sides, we have (4 AB cos θ) = 0

$\Rightarrow$ cos θ = 0 or θ = 90^o

Hence, the two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

61. Applying triangle rule of vector addition in the triangle ABC

$\overrightarrow{\mathbf{AC}} = \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}}$ ..... (i)

Similarly, in triangle ADE and triangle ACD we get

$\overrightarrow{\mathbf{DA}} = \overrightarrow{\mathbf{DE}} + \overrightarrow{\mathbf{EA}}$ ..... (ii)

$\overrightarrow{\mathbf{AC}} = \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{AD}}$ ..... (iii)

Therefore,

$\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DE}} + \overrightarrow{\mathbf{EA}}$

Let us group the above expression,

$(\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}}) + \overrightarrow{\mathbf{CD}} + (\overrightarrow{\mathbf{DE}} + \overrightarrow{\mathbf{EA}})$

$= \overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DA}}$

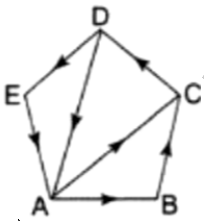
Substituting equation (i)

$= \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{DA}}$

$= \overrightarrow{\mathbf{AD}} - \overrightarrow{\mathbf{AD}}$

= 0

Hence, proved



62. $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$

$A = \sqrt{3^2 + 2^2 + 1^2}$

$\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k} = \sqrt{1^2 + 3^2 + 5^2}$

$B = \sqrt{35}$

$A^2 + C^2$ = 14 + 21 = 35

B2 = 35

So $A^2 + C^2 = B^2$

As it obey pyhthagorus theorem, so these vectors form right angle triangle.

63. **Polygon law of vector addition.** If a number of vectors are represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant is represented both in magnitude and direction by the closing side of the polygon taken in opposite order.

64. $F_1 = P + Q$

$F_2 = P - Q$

and $R = \sqrt{3P^2 + Q^2}$

Now, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

or, $R^2 = F_1^2 + F_2^2 + 2\,F_1\,F_2\cos\theta$

or, $3P^2 + Q^2$

$= (P + Q)^2 + (P - Q)^2 + 2(P + Q)\,(P - Q)\cos\theta$

$= P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ + 2(P^2 - Q^2)\cos\theta$

$= 2P^2 + 2Q^2 + 2(P^2 - Q^2)\cos\theta$

or, $\cos\theta = \frac{P^2 - Q^2}{2\left(P^2 - Q^2\right)} = \frac{1}{2}$ $= \cos 60^{o}$

$\theta = 60^{o}$

Thus, the angle of inclination between the forces must be 60^{o}.

65. Yes, because $\vec{a} + \vec{b}$ is represented by the diagonal of the parallelogram drawn with $\vec{a}$ and $\vec{a}$ as adjacent sides. The diagonal passes through the common tail of $\vec{a}$ and $\vec{b}$. However, $\vec{a} - \vec{b}$ is represented by the other diagonal of the same parallelogram not passing through the common tail of $\vec{a}$ and $\vec{b}$. Thus both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ lie in plane of the same parallelogram.

66. $|\vec{a} - \vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$

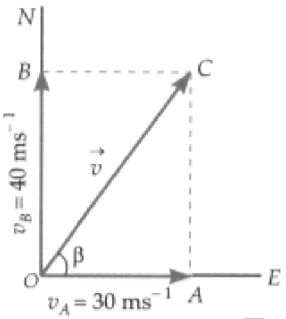
$= -4|\vec{a}||\vec{b}|\cos^2\frac{\theta}{2} = \text{a}$ negative quantity

Hence $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

67. Let the body start moving shown in Figure.

$\vec{v}_A = \overrightarrow{OA}$ = 30 ms^{-1}, due east

$\vec{v}_B = \overrightarrow{OB}$ = 40 ms^{-1}, due north



According to parallelogram law, $\overrightarrow{OC}$ is the resultant velocity.

Its magnitude is

$v = \sqrt{v_A^2 + v_B^2} = \sqrt{30^2 + 40^2} = 50\ \text{ms}^{-1}$

Suppose the resultant velocity $\vec{v}$ makes angle β with the east direction. Then

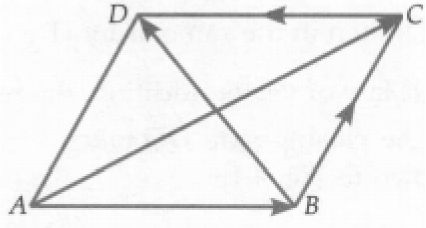
$\tan\beta = \frac{CA}{OA} = \frac{40}{30}$ = 1.3333

$\therefore \beta$ = $\tan^{-1}$ (1.3333) = $53^{o}8'$

68. Using triangle law of vector addition in figure,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$



i. $\overrightarrow{AC}+\overrightarrow{BD}=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{BC}+\overrightarrow{CD}$

But $\overrightarrow{AB}=-\overrightarrow{CD}$

$\therefore \overrightarrow{AC}+\overrightarrow{BD}=-\overrightarrow{CD}+2\overrightarrow{BC}+\overrightarrow{CD}=2\overrightarrow{BC}$

ii. $\overrightarrow{AC}-\overrightarrow{BD}=\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{BC}-\overrightarrow{CD}$

$=\overrightarrow{AB}-\overrightarrow{CD}=\overrightarrow{AB}-(-\overrightarrow{AB})=2\overrightarrow{AB}$

69. The resultant of two vectors is the vector sum of two vectors

$\vec{R}=\vec{a}+\vec{b}$

= $3\hat{i}+4\hat{j}+5\hat{k}+5\hat{i}+3\hat{j}+4\hat{k}$

= $8\hat{i}+7\hat{j}+9\hat{k}$ ...(1)

and magnitude of R $=|\vec{R}|=\sqrt{8^2+7^2+9^2}=\sqrt{64+49+81}$

= $\sqrt{194}$ = 13.9

now using the formula to find the angle between two vectors

$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ (formula of a vector)

$\cos\theta=\frac{(8\hat{i}+7\hat{j}+9\hat{k})\cdot\hat{i}}{|(8\hat{i}+7\hat{j}+9\hat{k}||\hat{i}|}=\frac{8}{13.9\times1}$

$\cos\theta$ = 0.57

θ = $\cos^{-1}(0.57)$

70. Given, $\vec{A}=\vec{B}-\vec{C}$ and $\vec{C}=\vec{B}-\vec{A}$

$\vec{C}\cdot\vec{C}=(\vec{B}\cdot\vec{A})\cdot(\vec{B}-\vec{A})$

$=\vec{B}\cdot\vec{B}-\vec{A}\cdot\vec{B}-\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{A}$

or $C^2=B^2-2\vec{A}\cdot\vec{B}+\vec{A}$

= $A^2+B^2-2AB\cos\theta$

Where θ is the angle between $\vec{A}$ and $\vec{B}$

or $2AB\cos\theta = A^2+B^2-C^2$

or $\cos\theta=\frac{A^2+B^2-C^2}{2AB}$

or $\theta\cos^{-1}\left(\frac{A^2+B^2-C^2}{2AB}\right)$

Section C

71. As shown in figure, the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are cyclic therefore

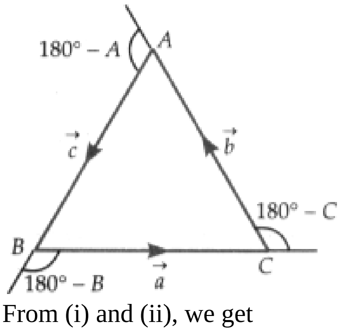
$\vec{a}+\vec{b}+\vec{c}=0$ or $\vec{a}+\vec{b}=-\vec{c}$

or $(\vec{a}+\vec{b})\times\vec{c}=-\vec{c}\times\vec{c}$ or $\vec{a}\times\vec{c}+\vec{b}\times\vec{c}=\vec{0}$

or $-\vec{c}\times\vec{a}+\vec{b}\times\vec{c}=\vec{0}$

or $\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Similarly, $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$



From (i) and (ii), we get

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

or $|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$

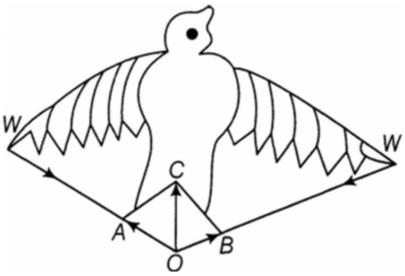
or ab sin (180° - C) = bc sin (180° - A)

= ca sin (180° - B)

or ab sin C = bc sin A = ca sin B

Dividing throughout by abc, we get
$\frac{\sin C}{c}=\frac{\sin A}{a}=\frac{\sin B}{b}$ or $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

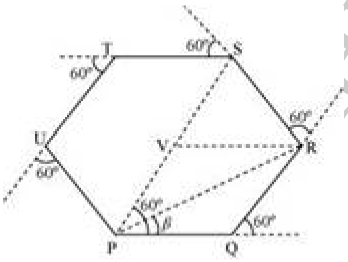
72. Yes, the flight of a bird is an example of composition of vectors. As the bird flies, it strikes the air with its wings W, W along WO. According to Newton's third law of motion, air strikes the wings in opposite directions with the same force in reaction. The reactions are $\overrightarrow{OA}$ and $\overrightarrow{OB}$. From law of parallelogram vectors, $\overrightarrow{OC}$ is the resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB}$. This resultant upwards force $\overrightarrow{OC}$ is responsible for the flight of the bird.



73. i. The addition of two scalar quantities is meaningful only if they both represent the same physical quantity.
ii. The addition of a vector quantity with a scalar quantity is not meaningful.
iii. A scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.
iv. A scalar, irrespective of the physical quantity, can be multiplied with another scalar having the same or different dimensions.
v. The addition of two vector quantities is meaningful only if they both represent the same physical quantity.
vi. A component of a vector can be added to the same vector as they both have the same dimensions.

74. i. Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes.
Hence, his net displacement is zero.
ii. Average velocity is given by the relation: Average velocity $=\frac{\text{net displacement}}{\text{total time interval}}$
Since the net displacement of the cyclist is zero, his average velocity will also be zero.
iii. Average speed of the cyclist is given by the relation: Average speed $=\frac{\text{total path length}}{\text{time interval}}$
Total path length = OP + PQ + QO
Total path length $=1+\frac{1}{4}(2\pi\times1)+1=2+\frac{1}{2}\pi=3.570\mathrm{km}$
Time taken $=10\mathrm{min}=\frac{10}{60}=\frac{1}{6}h$
$\therefore$ Average speed $=\frac{3.570}{\frac{1}{6}}=21.42\mathrm{km/h}$

Section D

75. 

The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure.
Let the motorist start from point P. The motorist takes the third turn at S.
Magnitude of displacement = PS = PV + VS = 500 + 500 = 1000 m ($\because PV=QR, VS=SR$)
Total path length, d_1 = PQ + QR + RS = 500 + 500 + 500 = 1500 m
The motorist take the sixth turn at point P, which is the starting point
$\therefore$ Magnitude of displacement = 0
Total path length, d_2 = PQ + QR + RS + ST + TU + UP
d_2 = 500 + 500 + 500 + 500 + 500+500 = 3000 m
The motorist takes the eight turn at point R
$\therefore$ Magnitude of displacement = PR
$PR=\sqrt{PQ^2+QR^2+2(PQ)\cdot(QR)\cos60^\circ}$
$PR=\sqrt{500^2+500^2+(2\times500\times500\times\cos60^\circ)}$

$PR = \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$

$PR = \ 866.03\ m$

$\beta = \tan^{-1}\left(\frac{500 \sin 60^\circ}{500 + 500 \cos 60^\circ}\right) = 30^\circ$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30^o with PR.

Total path length = Circumference of the hexagon + PQ + QR

Total path length= $6 \times 500 + 500 + 500 = 4000$ m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	**Magnitude of displacement (m)**	**Total path length (m)**
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30^o	4000