Solution

WORK ENERGY POWER IMP QUESTIONS

Class 11 - Physics

Section A

1. **(a)** 742.5 W

Explanation:

Total power required to overcome a force of 165 N and to maintain a speed of 9.00 m/s

F = 165N

v = 9m/s

P = Fv = 165 $\times$ 9 = 1485W

if each rider contribute equal power, then the power required per rider will be $\frac{P}{2}$ = $\frac{1485}{2}$ = 742.5W

2.

(c) 36%

Explanation:

Initial K.E., K = $\frac{p^2}{2m}$

Decrease in momentum

= 20% of p = $\frac{20}{100} \times p = \frac{p}{5}$

Final momentum = p - $\frac{p}{5} = \frac{4p}{5}$

Final K.E., $K' = \frac{(4p/5)^2}{2m} = \frac{16}{25}\frac{p^2}{2m} = \frac{16}{25}K$

% Decrease in K.E. = K - K' = K - $\frac{16}{25}K = \frac{9}{25}K$

= $\frac{K-K'}{K} \times$ 100 = $\frac{\frac{9}{25}K}{K} \times$ 100 = 36%

3.

(b) $\frac{K}{2}$

Explanation:

Initial kinetic energy, K = $\frac{1}{2}$ mu^2

Velocity at the highest point = Horizontal component of u

= u cos 45^o = $\frac{u}{\sqrt{2}}$

Hence K.E. at the highest point,

$K' = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2} \cdot \frac{1}{2}$ mu^2 = $\frac{K}{2}$

4.

(c) (2t^3 + 3t^5) W

Explanation:

$\vec{F}$ = (2t$\hat{i}$ + 3t$^2\hat{j}$) N

$\vec{a} = \frac{F}{m} = \frac{\left(2t\hat{i}+3t^2\hat{j}\right)\text{N}}{1\text{ kg}}$

$\frac{d\vec{v}}{dt}$ = (2t$\hat{i}$ + 3t$^2\hat{j}$) $\frac{m}{s^2}$

$\int_0^{\vec{v}} d\vec{v} = \int_0^t$ (2t$\hat{i}$ + 3t$^2\hat{j}$) dt

$\Rightarrow \vec{v}$ = (t$^2\hat{i}$ + 3t$^2\hat{j}$) $\frac{m}{s}$

$\vec{P} = \vec{F} \cdot \vec{v}$ = (2t$\hat{i}$ + 3t^2) $\cdot$ (t$^2\hat{i}$ + t$^3\hat{j}$)

= (2t^3 + 3t^5) W

5. **(a)** 360000 J, 101 m/s
Explanation:
Energy = P $\times$ Time = 100 $\times$ 1Hr
Energy = 100 $\times$ 1 $\times$ 60 $\times$ 60 = 360000 J
for a 70 Kg man $K = \frac{1}{2}mv^2$
speed of man v = $\sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times360000}{70}}$ = 101m/s

6.
(c) 10.36 m/s
Explanation:
Mass of trolley M = 200Kg
mass of child m = 20Kg
speed of trolley v = 36Km/hr = 36 $\times$ $\frac{5}{18}$ = 10m/s
Let v' be the final velocity of the trolley with respect to the ground.
The final velocity of the boy with respect to the ground = v' - 4
from conservation of linear momentum
$p_i = p_f$
(M + m)v = Mv' +m(v' − 4)
(200 + 20) $\times$ 10 = 200v' + 20(v' − 4)
2200 = 220v' - 80
= $\frac{2280}{220}$ = 10.36m/s

7.
(c) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
Explanation:
By conservation of energy, Final K.E. = Initial K.E. - Excitation energy
$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \varepsilon$

8.
(b) work done by F(x).
Explanation:
Work done by a variable force is given by
W = $\int F(x)dx$
The above integration gives us the area under the curve F(x) Vs x.

9.
(b) potential energy is minimum
Explanation:
In the position of equilibrium, the potential energy of the simple pendulum is minimum.

10.
(c) $E_1 < E_2$
Explanation:
Kinetic energy E = $\frac{p^2}{2m}$
For same p, = E $\propto$ $\frac{1}{m}$
$\therefore \frac{E_1}{E_2} = \frac{m_2}{m_1}$
As $m_1 > m_2$
So, $E_1 < E_2$

11.

(d) zero

Explanation:

Motion without slipping implies pure rolling. During pure rolling work done by the frictional force is zero.

12.

(b) 1.92 J

Explanation:

$m_1v_1 = m_2v_2$

$3 \times 1.6 = 6 \times v_2$

$\Rightarrow v_2 = 0.8ms^{-1}$

K.E. of 6 kg mass $= \frac{1}{2} \times 6 \times (0.8)^2$ J

= 1.92 J

13.

(c) 6 J

Explanation:

$W = \int F\,dx = \int_0^2 (x + x^3)\,dx = \left[\frac{x^2}{2} + \frac{x^4}{4}\right]_0^2$

= 6 J

14. **(a)** none of these

Explanation:

$W = Fs\cos\theta$

$25 = 10 \times 10\cos\theta$

$\cos\theta = \frac{1}{4}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{4}\right)$

15.

(c) remains constant

Explanation:

By conservation of momentum,

$m_1v + m_2 \times 0 = (m_1 + m_2)v'$

$\therefore v' = \frac{m_1}{m_1+m_2} \cdot v$

As $\frac{m_1}{m_1+m_2} < 1$

so, $v' > v$

Hence velocity of the system decreases at the time of collision.

16.

(b) 16U

Explanation:

$U = \frac{1}{2}kx^2$

$\therefore \frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$

$U_2 = 16U_1 = 16U$

17.

(b) 1 m/s to the left

Explanation:

from conservation of linear momentum initial momentum = final momentum

$\vec{p}_i = \vec{p}_f$

$(500 \times 0) + [500 \times (-2)] = (500 + 500)v$
$- 1000 = 1000v$
v = −1m/s negative sign indicates that cars move to the left.

18.

(b) 4.8 kJ
Explanation:
By conservation of momentum,
$MV = m_1v_1 + m_2v_2$
$3 \times 0 = 2 \times v_1 + 1 \times 80$
$v_1 = -40\ ms^{-1}$
Total energy imparted to the two fragments
$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
$= \frac{1}{2} \times 2 \times (40)^2 + \frac{1}{2} \times 1 \times (80)^2$
= 4800 J = 4.8 kJ

19.

(d) Zero
Explanation:
As the road does not move at all, therefore, work done by the cycle on the road is zero.
W = 0, because road cannot move. So, s = 0
$W = F \times s = F \times 0 = 0J$

20.

(b) 200 J
Explanation:
$W = \mu mg = 0.2 \times 5 \times 10 \times 20 = 200$ J

21.

(d) 6.25×10^4 N/m
Explanation:
The kinetic energy of the car will be converted into the potential energy of spring.
$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$
$k = \frac{mv^2}{x^2}$
m = 1200Kg
v = 0.65m/s
x = 0.09m
$k = \frac{1200 \times 0.65 \times 0.65}{0.09 \times 0.09} = 6.25 \times 10^4$N/m

22.

(b) $t^{3/2}$
Explanation:
We know that
Power, P = Fv = k (constant)
Considering the dimensions,
[P] = [F][v]
$\Rightarrow [P] = [MLT^{-2}][LT^{-1}]$
$\Rightarrow [P] = [ML^2T^{-3}]$
So, $[ML^2T^{-3}] = k$

$\Rightarrow [L^2T^{-3}] = k$ ...[M is constant]

$\Rightarrow [L^2/T^3] = k$

$\Rightarrow [L^2] = k[T^3]$

$\Rightarrow [L^2] \propto [T^3]$

$\Rightarrow [L] \propto [T^{3/2}]$

Hence, $L \propto t^{3/2}$

23.

(c) $\frac{3}{5}$v

Explanation:

Mass of an α-particle is four times the mass of a neutron

$\therefore m_1 = m, m_2 = 4m,$

$u_1 = v, u_2 = 0, v_1 = ?$

$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

$= \frac{m-4m}{m+4m} \cdot v + 0 = -\frac{3}{5} v$

24. **(a)** Statement (iv) is correct.

Explanation:

Work done by weight-lifter is zero because there is no displacement. In a locomotive, work done is zero because force and displacement are mutually perpendicular to each other.

While a person holding a suitcase, work done is zero because there is no displacement.

25.

(b) Momentum

Explanation:

In an inelastic collision, momentum is conserved and K.E. is not conserved.

26.

(c) 2 E_A

Explanation:

$E = \frac{1}{2} kx^2 = \frac{1}{2} \frac{F^2}{k}$

For same F, $E \propto \frac{1}{k}$

$\therefore \frac{E_B}{E_A} = \frac{k_A}{k_B} = \frac{2k_B}{k_B} = 2$ or $E_B = 2 E_A$

27.

(b) $\frac{CX_1^2}{2}$

Explanation:

$W = \int F\, dX = \int_0^{X_1} CX\, dX = C\left[\frac{X^2}{2}\right]_0^{X_1} = \frac{1}{2} C X_1^2$

28.

(b) 8.5 cm

Explanation:

For maximum compression of spring, kinetic energy will be converted into the potential energy of spring.

$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$x^2 = \frac{mv^2}{k} = \frac{6 \times 3 \times 3}{75 \times 10^2}$

$x = \sqrt{\frac{6 \times 3 \times 3}{75 \times 10^2}}$ = 0.085m = 8.5cm

29.

(b) 300 J

Explanation:

$U_1 = \frac{1}{2}kx_1^2$ = 100 J

$U_2 = \frac{1}{2}kx_2^2$

$\frac{U_2}{U_1} = \frac{x_2^2}{x_1^2} = \left(\frac{4}{2}\right)^2$ = 4

$U_2 = 4U_1 = 400$ J

$\therefore U_2 - U_1 = 300$ J

30.

(d) 576 mJ

Explanation:

$x = 3t - 4t^2 + t^3$

$v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$a = \frac{dv}{dt} = -8 + 6t$

At t = 4s,

$a = -8 + 6 \times 4 = 16$ ms^{-2}

x = 12 - 64 + 64 = 12 m

Now, m = 3g = 3×10^{-3} kg

W = Fx = max = $3 \times 10^{-3} \times 16 \times 12$

= 576×10^{-3} J = 576 mJ

Section B

31. Power, P = Fv = ma $\cdot$ v = m$\frac{dv}{dt}\cdot$ v

or vdv = $\frac{P}{m}$ dt

Integrating both sides, we get

$\int$vdt = $\int \frac{P}{m}$ dt = $\frac{P}{m}\int$ dt [$\because$ P, m are constant]

or $\frac{v^2}{2} = \frac{P}{m}\cdot t$

or $v^2 = \frac{2P}{m}\cdot t$

or v = $\sqrt{\frac{2P}{m}}\cdot t^{\frac{1}{2}}$

Thus v $\propto t^{\frac{1}{2}}$

Also v = $\frac{dx}{dt}$ or dx = vdt = $\sqrt{\frac{2P}{m}}\cdot t^{\frac{1}{2}}$ dt

Integrating both sides, we get,

$\int$ dx = $\sqrt{\frac{2P}{m}}\int t^{\frac{1}{2}}$ dt,

or x = $\sqrt{\frac{2P}{m}}\cdot\frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2P}{m}}\cdot t^{\frac{3}{2}}$

Thus x $\propto t^{\frac{3}{2}}$

32. Pressure = 130 mm of Hg = 13 cm of Hg

= hρg = $13 \times 1.03 \times 980$ dyne cm^{-2}

Volume = 4000 cm^{-3}; Time, t = 1 min = 60 s

Power, P = $\frac{W}{t} = \frac{\text{Pressure} \times \text{volume}}{t}$

$= \frac{13\times1.03\times980\times4000}{60}$

= 8748133.33 erg s^{-1}

= $8748133.33 \times 10^{-7}$ Js^{-1} or W

= $\frac{874813.33\times10^{-7}}{746}$ hp = 1.17×10^4 hp

33. If a force F applied to a wire increases its length by x, then accordingly to Hooke's law, F = kx, where k is the force constant.

i. Given F = mg = 5 $\times$ 10 = 50 N

x = 0.5 cm = 0.5 $\times$ 10^{-2} m

$\therefore$ k = $\frac{F}{x} = \frac{50}{0.5\times10^{-2}}$ = 1.0 $\times$ 10^4 Nm^{-1}

ii. Work done in stretching the wire,

W = $\frac{1}{2}$ kx^2 = $\frac{1}{2}$ $\times$ 1.0 $\times$ 10^4 $\times$ $(0.5 \times 10^{-2})^2$ = 0.125 J

34. Here m_1 = M, u_1 = v, m_2 = m, u_2 = 0

Velocity of M after collision,

$v_1 = \frac{m_1-m_2}{m_1+m_2}$ u_1 + $\frac{2m_2}{m_1+m_2}\cdot$ u_2

$= \frac{M-m}{M+m}\cdot$ v + $\frac{2m}{M+m}\cdot 0$

Kinetic energy lost by mass M

= $\frac{1}{2}$ Mv^2 - $\frac{1}{2}M\left(\frac{M-m}{M+m}v\right)^2$

= $\frac{1}{2}$ $Mv^2\left[1-\left(\frac{M-m}{M+m}\right)^2\right]$

= $\frac{1}{2}$ $Mv^2 \cdot \frac{4Mm}{(M+m)^2}$

= $\frac{1}{2}$ $Mv^2 \cdot \frac{4Mm}{M^2}$ [M + m $\approx$ M]

= 2 mv^2

35. By work-energy theorem,

W = P $\times$ t = $\frac{1}{2}$ mv^2

or $v^2 = \frac{2Pt}{m}$

$\therefore$ v = $\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{\frac{1}{2}}$

On integration,

s = $\left(\frac{2P}{m}\right)^{\frac{1}{2}} \frac{2}{3} t^{\frac{3}{2}}$

$\therefore$ s $\propto t^{\frac{3}{2}}$

Hence, alternative (iii) is correct.

36. For the car :

m_1 = 1000 kg, u_1 = 32 ms^{-1}, v_1 = -8 ms^{-1}

For the truck: m_2 = 8000 kg, u_2 = 4 ms^{-1}, v_2 = ?

By conservation of linear momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

1000 $\times$ 32 + 8000 $\times$ 4 = 1000 $\times$ (- 8) + 8000 v_2

or 64000 + 8000 = 8000 v_2

or $v_2 = \frac{72000}{8000}$ = 9 ms^{-1} (in the same direction)

37. Change in total energy of block = Work done against friction

$\therefore$ mg (h - x) + $\frac{1}{2}$ m $\left(v_A^2 - v_B^2\right)$ = fs

3 $\times$ 9.8 $\times$ (4 - 1) + $\frac{1}{2}$ $\times$ 3 $\times$ $(2^2 - 6^2)$ = f $\times$ 12

9 $\times$ 9.8 - 3 $\times$ 16 = f $\times$ 12

or f = $\frac{40.2}{12}$ = 3.35 N

Total energy at B = 3 $\times$ 9.8 $\times$ 1 + $\frac{1}{2}$ $\times$ 3 $\times$ 6^2

= 29.4 + 54 = 83.4

This energy is used in doing work against friction.

$\therefore$ f $\times$ s' = 83.4 or $s' = \frac{83.4}{f} = \frac{83.4}{3.35}$ = 24.5 m

38. Let the initial and final velocities of 30 quintal body be u and v and the velocities of 90 quintal body be u' and v' respectively.

It is based on the laws of conservation of energy and momentum so,

mu + m'u' = mv + m'v' ...(i)

and law of conservation of (kinetic) energy

now, here
Mass(m) = 30 quintals = 3000 kg
mass (m') = 90 quintals = 9000 kg
u = 18 km/hr = 5 m/s
u' = 14.4 km/hr = 4 m/s
thus, equation (i) will be as following
$3000 \times 5 + 9000 \times 4 = 3000$ v + 9000 v' on dividing by 3000 on both sides
5 + 12 = v + 3 v'
or
v + 3v' = 17 ...(iii)
or v = 17 - 3v' ...(iv)
so, equation (2) will be
$3000 \times 52 \times 90000 \times 42 = 3000v^2 + 9000v^2$
$25 + 48 = v^2 + 3v^2$
or
$v^2 + 3v^2 = 73$
now, by substituting the value of v from (iv) in (v), we get
$(17 - 3v')^2 + 3v'^2 = 73$
or
$12v'^2 - 102v' + 216 = 0$
dividing both sides by 6, we have
$2v'^2 - 17v' + 36 = 0$
now, by solving the above quadratic equation given us two values of v' that is
v' = 4 m/s or v' = 4.5 m/s
we can substitute both value of v; one by one in (iii) to get two corresponding value of v

a. if v' = 4m/s ; v = 5 m/s and
b. if v' = 4.5 m/s = 3.5 m/s

we shall select option (b) to be the correct values as option (a) represents the initial velocities. So, the final velocities will be v = 3.5 m/s and v' = 4.5 m/s

39. As the elevator is moving down with a uniform speed (a = 0), so the value of g remains the same.
Here m = 0.3kg, h = 3 m, g = $9.8ms^{-2}$
P.E. lost by the bolt = mgh = $0.3 \times 9.8 \times 3 = 8.82$ J
As the bolt does not rebound, the energy is converted into heat.
$\therefore$ Heat produced = 8.82 J
Even if the elevator were stationary, the same amount of heat would have produced because the value of g is the same in all inertial frames of reference.
40. Momentum of the bullet = $10^{-2} \times 2 \times 10^2 = 2$ kg ms^{-1}
Let the combined velocity of the bob + bullet = v
Momentum of the bob + bullet
$= (10^{-2} + 1)v = 1.01v$
By conservation of momentum, 1.01 v = 2 kg ms^{-1}
or v = $\frac{2}{1.01}$ = 1.98 ms^{-1}
By conservation of energy,
$\frac{1}{2}$ (M + m) v^2 = (M + m)gh
or h = $\frac{v^2}{2g} = \frac{(1.98)^2}{2\times10}$ = 0.196 m $\approx$ 0.2 m
41. Let us assume that masses of both the bearings are same, i.e.
$m_1 = m_2 = m$
$u_1 = v, u_2 = -v$
We have taken u_1, = v and u_2 = -v because both the ball bearings are moving with same velocity but in opposite directions.

As the collision is perfectly elastic, velocities after the collision will be given by the following equations,

$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

$= \frac{m - m}{m + m} \cdot v + \frac{2m}{m + m}(-v) = 0 - v = -v$

$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

$= \frac{2m}{m + m} \cdot v + \frac{m - m}{m + m} \cdot (-v) = v + 0 = v$

Thus, the two ball bearings bounce back with equal speed after the collision.

42. a. When the velocity of the aeroplane is doubled, its momentum also gets doubled. However, the combined momentum of aeroplane + air is conserved. As the momentum of the aeroplane increases, the momentum of air also increases by an equal amount in the opposite direction.

    b. The kinetic energy becomes four times. The additional energy is obtained by burning of fuel. However, the total energy is still conserved.

43. In the given problem the nature of the force is variable and hence we will use the following equation to calculate the work done by the force

    ∫ dW = ∫ Fdx

    $W = \int_0^5 F \cdot dx = \int_0^5 \left(7 - 2x + 3x^2\right) dx$

    $= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5$

    $= 7(5 - 0) - (5^2 - 0) + (5^3 - 0)$

    = 135 J

44. Total energy of the particle, E = 1J

    Force constant, k = 0.5 N m^{-1}

    Kinetic energy of the particle, $\mathrm{K} = \frac{1}{2}mv^2$

    According to the conservation law:

    E = V + K

    $1 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

    At the moment of ‘turn back', velocity (and hence K) becomes zero.

    $\therefore 1 = \frac{1}{2}kx^2$

    $\frac{1}{2} \times 0.5x^2 = 1$

    $x^2 = 4$

    $x = +2$

    Hence, the particle turns back when it reaches $x = \pm 2$ m.

45. Here m_1 = 10 kg, m_2 = 20 kg, u_1 = 20 ms^{-1}, u_2 = -10 ms^{-1}

    $v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

    $= \frac{10 - 20}{10 + 20} \times 20 + \frac{2 \times 20}{10 + 20} \times (-10)$

    $= -\frac{20}{3} - \frac{40}{3} = -\frac{60}{3} = -20\ ms^{-1}$

    $v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

    $= \frac{2 \times 10}{10 + 20} \times 20 + \frac{20 - 10}{10 + 20} \times (-10)$

    $= \frac{40}{3} - \frac{10}{3} = \frac{20}{3} = 10\ ms^{-1}$

46. As the particle is moving in horizontal circle, so

    Centripetal force, $F = \frac{mv^2}{r} = \frac{K}{r^2}$

    This gives $mv^2 = \frac{K}{r}$

    $\therefore$ K.E. of the particle,

    $K = \frac{1}{2} mv^2 = \frac{K}{2r}$

    As $F = -\frac{dU}{dr}$

    $\therefore$ Potential energy,

    $U = -\int_{\infty}^{r} F dr = -\int_{\infty}^{r} \left(-\frac{K}{r^2}\right) dr = K \int_{\infty}^{r} r^{-2} dr = -\frac{K}{r}$

    Total energy = $K + U = \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$

47. Here m = 0.4 kg, u = 9.8 ms^{-1}, a = -g = -9.8 ms^{-2}, t = $\frac{1}{2}$ s

i. s = ut + $\frac{1}{2}$ at^2 = 9.8 $\times \frac{1}{2} - \frac{1}{2} \times$ 9.8 $\times \frac{1}{2} \times \frac{1}{2}$ = 3.67 m

$\therefore$ P.E. = mgh = 0.4 $\times$ 9.8 $\times$ 3.67 = 14.386 J

ii. v = u + at = 9.8 - 9.8 $\times \frac{1}{2}$ = 4.9 ms^{-1}

$\therefore$ K.E. = $\frac{1}{2}$ mv^2 = $\frac{1}{2}$ $\times$ 0.4 $\times$ 4.9 $\times$ 4.9 = 4.802 J

48. To raised a body up to height h a person will apply the force upward in the direction of displacement

a. Work done by force is positive

$WD = \overline{F}d\overline{s} = Fds\cos\theta$

$\Rightarrow Fds\cos 0^\circ = Fdsis + ive \quad \theta = 0^\circ$.

b. Gravitational force is always downward, but displacement is upward here. So $\theta = 180^\circ$

$\therefore$ WD = $Fds\cos 180^\circ = -Fds$

Hence, WD by gravitational force is negative.

49. Here m_1 = 5 kg, u_1 = 5 ms^{-1}, m_2 = 3 kg, u_2 = 3 ms^{-1}

$\therefore v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

$= \frac{5-3}{5+3} \times 5 + \frac{2\times 3}{5+3} \times 3$

= $\frac{2}{8} \times 5 + \frac{6}{8} \times 3 = \frac{5}{4} + \frac{9}{4} = \frac{14}{4}$ = 3.5 ms^{-1}

$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

$= \frac{2\times 5}{5+3} \times 5 + \frac{3-5}{5+3} \times 3$

$= \frac{50}{8} - \frac{6}{8} = \frac{44}{8}$ = 5.5 ms^{-1}

50. a. Here s = 0, so W = Fs cos θ = 0

b. Here θ = 90^o, so W = Fs cos θ = 0

51. Percentage increase in K.E.,

(E) = $\left[\frac{\mathbf{E_2 - E_1}}{\mathbf{E_1}}\right] \times 100 = \left[\frac{\mathbf{E_2}}{\mathbf{E_1}} - \mathbf{1}\right] \times 100$

But Eap^2 $\Rightarrow \frac{\mathbf{E_2}}{\mathbf{E_1}} = \frac{\mathbf{p_2^2}}{\mathbf{p_1^2}}$

$\therefore$ % increase in K.E. = $\left[\frac{p_2^2}{p_1^2} - 1\right] \times 100$

Let p_1 = 100, then

p_2 = 100 + 50 = 150% increase in

K.E. = $\left[\frac{(150)^2}{(100)^2} - 1\right] \times 100 = \left[\frac{225}{100} - 1\right] \times 100$

= 125%

Therefore, the percentage increase in its kinetic energy is 125%.

52. In figure (a), the man carries the mass of 15 kg on his hands and walks 2 m. In this case, he is actually doing work against the friction force.

Friction force contribution by mass

$f = \mu N = \mu mg \times 15 \times 9.8$N

and work done against friction

W_1 = fs = $\mu \times 15 \times 9.8 \times 2 = 294\mu J$

In figure (b) the tension in the string, T = mg = 15$\times$ 9.8 N Hence, force applied by man for pulling the rope

F = T = 15 $\times$ 9.8 N

$\therefore$ Work done by man, W_2 = fs = 15$\times$ 9.8 $\times$ 2 = 294 J and additional work has to be done against friction also. Thus, it is clear that $W_2 > W_1$.

53. We know that, work done is given by W = Fs cosθ

In first case, angle (θ) between F and s is 90° because weight of the bag, i.e. force acts perpendicular to the displacement.

Therefore, W = Fs cos 90°= zero

In second case, F and s are in the same direction, i.e, θ = 0°.

so, F = μ mg

= 0.1$\times$ 20$\times$9.8 = 19.6 N

$\therefore$ W = $19.6 \times 10 \times \cos 0 = 196\text{J}$

So, work done in second case is more.

54. Since Neutron collides with the nucleus, momentum before the collision = u(m_1 - m_2) where u is the initial velocity of the protons = 10^6 m/s.

$\therefore$ Momentum = 10^6(m_1 - m_2)

After collision, let the speed of the Neutron after the collision be v.

$\therefore$ Momentum after collision = v(m_1 + m_2)

Now, Momentum will remains conserved during the collision,

$\therefore$ u(m_1 - m_2) = v(m_1 + m_2)

$\Rightarrow$ 10^6(1 - 80) = v(1 + 80) [Mass of the Proton = 1 u].

$\therefore$ v = $\frac{10^6(1-80)}{(1+80)}$

$\Rightarrow$ $\frac{v}{u}$ = $\frac{(1-80)}{(1+80)}$...eq(i).

Now, Fraction of the Kinetic energy retained = $\frac{0.5mv^2}{0.5mu^2}$

$\Rightarrow$ $\frac{0.5mv^2}{0.5mu^2} = \left(\frac{v}{u}\right)^2$

$\Rightarrow$ $\frac{v^2}{u^2}$ = $\left[\frac{(1-80)}{(1+80)}\right]^2$

$\Rightarrow$ $\left(\frac{v}{u}\right)^2$ = $\frac{(-79)^2}{(81)^2}$

$\Rightarrow$ $\left(\frac{v}{u}\right)^2$ = $\frac{6241}{6561}$

$\therefore$ Fraction of the energy retained by the Nucleus is $\frac{6241}{6561}$.

55. Here

m = 6.5 metric ton = 6500 kg, g = 9.8 ms^{-2},

v = 9 $kmh^{-1} = 9 \times \frac{5}{18} = 2.5\ \text{ms}^{-1}, \sin\theta = \frac{5}{13}$

$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

Total force required against which the engine needs to work

F = mg sin θ + f = mg sin θ + μ mg cos θ

= mg (sin θ + μ cos θ)

$= 6500 \times 9.8 \left[\frac{5}{13} + \frac{1}{12} \times \frac{12}{13}\right]$ = 29400 N

Power of the engine = Fv = 29400 $\times$ 2.5 = 73500 W = 73.5 kW

56. Here, Since, he is caring the suitcase over his head, force is acting perpendicularly.

Since, work done (w) = Force (F) $\times$ (s) $\times$ cos (θ)

And the force is acting perpendicularly, so, θ = 90^o

$\therefore$ w = F $\times$ s $\times$ cos(90^o)

$\Rightarrow$ w = F $\times$ s $\times$ 0

$\because$ cos(90^o) = 0...

Thus, work done = 0 Joules

57. If m is the mass of the ball, then its total initial energy at height h

= $\frac{1}{2}$ mu^2 + mgh

Energy after collision = 50% of ($\frac{1}{2}$ mu^2 + mgh) = $\frac{1}{2}$($\frac{1}{2}$ mu^2 + mgh)

As the ball rebounds to height h, so,

$\frac{1}{2}$ ($\frac{1}{2}$ mu^2 + mgh) = mgh or $\frac{1}{4}$ mu^2 = $\frac{1}{2}$ mgh

u = $\sqrt{2gh} = \sqrt{2 \times 9.8 \times 10}$ = 14 ms^{-1}

58. Mass per unit length of the chain = $\frac{m}{l}$

Let the length of the hanging part of the chain = y

Mass of the hanging part of the chain = $\frac{m}{l} \cdot$ y

Force required to be applied = Weight of the hanging part

or F = $\left[\frac{m}{l}y\right] g = \frac{mg}{l} \cdot y$

The work done in pulling the chain through a small distance dy is

dW = $-\frac{mg}{l}\cdot$ y dy

Here negative sign indicates that the weight and displacement are oppositely directed. Total work done in pulling the $\frac{1}{n}$th length of the chain is

W = $\int$ dW = $-\frac{mg}{l}\int_{y=\frac{l}{n}}^{y=0} ydy$

$= -\frac{mg}{l}\left[\frac{y^2}{2}\right]_{l/n}^{0}$

$= -\frac{mg}{2l}\left[0-\frac{l^2}{n^2}\right] = \frac{mgl}{2n^2}$

59. a. Power consumed by the family is, P = 8000 W

Solar energy received is = 200 W/square meter

The efficiency of conversion is = 20 %

let area required to generate the desired electricity is = A

As per the information given in the question, we have:

Power consumed is given by = efficiency of conversion $\times$solar energy received per unit area$\times$Area

$8000 = \frac{20}{100}\times(A\times 200)$

$\therefore A = \frac{8\times 10^3}{40} = 200\mathrm{m}^2$

b. The area needed is comparable to roof area of a large sized house.

60. i. Suppose the two springs get stretched by distances x_1 and x_2 by the same force F. Then

F = $k_1x_1 = k_2x_2$

$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1x_1\cdot x_1}{k_2x_2\cdot x_2} = \frac{F\cdot x_1}{F\cdot x_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$

As $k_1 > k_2$

$\therefore W_1 < W_2$ or $W_2 > W_1$

ii. Suppose the two springs are stretched by the same distance x. Then

$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x^2}{\frac{1}{2}k_2x^2} = \frac{k_1}{k_2}$

As $k_1 > k_2$

$\therefore W_1 > W_2$

Section C

61. From the law of conservation of linear momentum, we have

$mu_1 + 0 = mv_1 + mv_2$

or $u_1 = v_1 + v_2$ ....(i)

By conservation of energy, we have

$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

or $u_1^2 = v_1^2 + v_2^2$

From Equation (i), we obtain ....(ii)

$u_1 \cdot u_1 = (v_1 + v_2)\cdot(v_1 + v_2)$

$= v_1 . v_1 + v_1 . v_2 + v_2 . v_1 + v_2 . v_2$

or $u_1^2 = v_1^2 + v_2^2 + 2\mathbf{v}_1\cdot\mathbf{v}_2$

or $u_1^2 = u_1^2 + 2\mathbf{v}_1\cdot\mathbf{v}_2$ [by using Equatio (ii)]

or $v_1 . v_2 = 0$

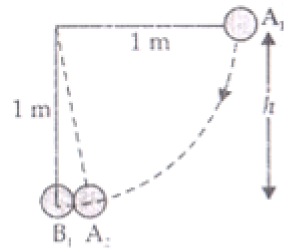
or $\theta_1 + \theta_2 = 90^\circ$

$\theta_1 = 90^\circ - \theta_2$

$= 90 - 37^0 = 53^0$

62. a. When a moving bob strike to an identical bob at rest then one bob transfers its momentum to other and itself become at rest. When bob A reaches at the position of bob B. The PE of A converts into KE where bob A transfers its momentum to bob B and bob A becomes itself in rest after elastic collision. So there will be no rise in bob A.

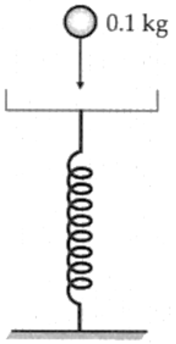
b. For speed of bob B



$P.E_{A_1} = K.E._{A_2} = K.E_{B_2}$
$\frac{1}{2}mv_3^2 = mgh$
$v_B = \sqrt{2g \times 1} = \sqrt{2 \times 9.8} = \sqrt{\frac{2}{10} \times 2 \times 49}$
$= 2 \times 7\frac{1}{\sqrt{10}} = \frac{14 \times \sqrt{10}}{10}$
$= 1.4 \times 3.16$
$v_B = 4.42 \text{m/s}$

63. Let V be the volume of water flowing out per second from the tub which is kept at height h from the ground. Then,
Input power = $(V\rho)$ gh [$\because V\rho$ = mass of water flowing per second]
Output power = 40 W
As efficiency,
$\eta = \frac{\text{Output power}}{\text{Input power}}$
$\therefore \frac{90}{100} = \frac{40}{V\rho gh}$
or V = $\frac{40 \times 100}{90\rho gh} = \frac{40 \times 100}{90 \times 1000 \times 9.8 \times 10}$
= 0.453 $\times$ 10^{-3} m^3/s = 0.453 litre/s [$\therefore$ 1 litre = 10^{-3} m^3]
Time for which the bulb can be kept on
= $\frac{200 \text{ litre}}{0.453 \text{ litre}/s}$ = 441 s.

64. Here m = 0.1 kg, h = 0.24 m, x = 0.01 m, h' = ?, x' = 0.04 m
As the particle is dropped from a height h, it compresses the spring through distance x



$\therefore$ Total loss in P.E. of the particle = Total gain in P.E. of the spring.
or mg (h + x) = $\frac{1}{2}$ kx^2 ...(i)
and mg $(h' + x')$ = $\frac{1}{2}$ kx'^2 ...(ii)
Dividing (ii) by (i), we get
$\frac{h'+x'}{h+x} = \left(\frac{x'}{x}\right)^2$
or $\frac{h'+0.04}{0.24+0.01} = \left(\frac{0.04}{0.01}\right)^2$
On solving , h' = 3.96 m

65. a. In an elastic collision of two billiard balls, the total kinetic energy is not conserved during the short time of collision of the balls. During this period, the kinetic energy of the balls is zero. All the energy is stored in the form of potential energy.
b. The total linear momentum is always conserved during the entire duration of collision. The individual linear momenta change but the total linear momentum always remains constant.
c. For an inelastic collision, kinetic energy is not conserved but the total linear momentum is conserved.
d. The collision is elastic in nature. The forces involved are conservative in nature since potential energy depends on the separation of the centres.

66. Initial kinetic energy of the body,

K = $\frac{1}{2}$ mv^2 = $\frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$

Increase in momentum = 20% of p = $\frac{20}{100}\times$ p = $\frac{p}{5}$

Final momentum = p + $\frac{p}{5} = \frac{6p}{5}$

Final kinetic energy of the body,

$K' = \frac{(6p/5)^2}{2m} = \frac{36}{25}\frac{p^2}{2m} = \frac{36}{25}K$

Increase in kinetic energy

= K' - K = $\frac{36}{25}$ K - K = $\frac{11}{25}$ K

% Increase in K.E.

$= \frac{K'-K}{K}\times$ 100 = $\frac{\frac{11}{25}K}{K}\times$ 100 = 44%.

67. Useful power developed = 5000 hp

Efficiency = 85%

$\therefore$ Total power generated

= $\frac{100}{85}\times$ 5000 hp = $\frac{100\times5000\times746}{85}$ W

Total work done by the falling water in 1 min or 60 s,

W = Pt = $\frac{100\times5000\times746}{85}\times$ 60 = 26.94 $\times$ 10^7 J

Now mgh = W

$\therefore$ m = $\frac{W}{gh} = \frac{26.94\times10^7}{10\times50}$ = 5.39 $\times$ 10^5 kg

68. a. Let the area needed to supply 8 kW = A m^2

Energy incident per unit area = 200 W

Energy incident on area A = 200 $\times$ A W

= 20% of 200 $\times$ A = 40 A W

But 40 A W = 8 kW = 8000 W

or A = $\frac{8000}{40}$ = 200 m^2

b. Area of the roof of the given house,

A' = 70% of 20m $\times$ 15m

= $\frac{70\times20\times15}{100}$ = 210 m^2

Required ratio = $\frac{A}{A'} = \frac{200}{210}$ = 20 : 21

69. i. For region A: V > E

E = V + K $\Rightarrow$ K = E - V

$\because$ V > E So K < 0

K.E. is negative. Which is not possible. Thus the particle cannot be in this region.

ii. In region B: V < E

$\Rightarrow$ K = E - V

$\because$ K > 0

This case is possible. Thus particle can be in this region.

iii. Region C:K > E as total energy E = K + V

$\Rightarrow$ V = E - K

$\therefore$ V < 0

P.E. is negative.

This is also possible because P.E. can be negative. Thus particle can be found in this region.

iv. Region D: V > K

K = E - V

This is also possible as P.E. for a system can be greater than KE.

Thus particle can be found in this region.

70. Given that

Radius of the well, R = $\frac{3}{2}$ m

Area of cross-section of the well

$= \pi R^2 = \frac{22}{7}\times\left(\frac{3}{2}\right)^2 = \frac{99}{14}$ m^2

Volume of the water in the well = Area of the cross-section $\times$ depth of well

= $\frac{99}{14}\times$ 14 = 99 m^3

$\therefore$ Mass of water in the well = Volume $\times$ density = 99 $\times$ 10^3 kg

As the well is emptied, the height through which water has to be raised by the engine changes from (20 - 14)m in the beginning to (20 - 0) m at the end.

$\therefore$ Average height raised = $\frac{6+20}{2}$ = 13 m

Work required to empty the well,

W = mgh = 99 $\times$ 10^3 $\times$ 9.8 $\times$ 13 = 12612600 J

Power, P = 5 hp = 5 $\times$ 746 W

Required time, t = $\frac{W}{P} = \frac{12612600}{5\times746}$ = 3381.6 s

71. Given, $_1H^2$ = 2.014 u, $_2He^3$ = 3.0160 u, Mn = 1.0087 u

1 u = 1.66 $\times$ 10^{-27} kg

Energy released = ?

mass of reactant, $mass_r$ = 2 $\times$ 2.0141 = 4.0282 u

mass of products, $mass_p$ = 3.0160 + 1.0087 = 4.0247 u

Loss of mass = $mass_p$ - $mass_r$

$\triangle$M = (4.0282 - 4.0247) u

= 0.0035 u

= 0.0035 $\times$ 1.66 $\times$ 10^{-27} kg

Energy released E = ($\triangle$m)c^2

E = 0.0035 $\times$ 1.66 $\times$ 10^{-27} $\times$ (3 $\times$ $10^8)^2$

= 52.2 $\times$ 10^{-14} J

As we know 1 MeV = 1.602 $\times$ 10^{-13} J

Energy released = $\frac{52.2\times10^{-14}}{1.602\times10^{-13}}$ = 3.26 MeV

72. i. Ball (1) is rolling down without slipping, so zero (0) force of friction acts or no loss of energy, hence, the total mechanical energy is conserved.
Ball 3 has negligible friction hence there is no loss of energy, so total mechanical energy is conserved. Hence, mechanical energy is conserved for ball (1) and (3).
ii. only ball 3 can reach D
iii. ball 1 and 2 cannot reach point A.

73. According to the question in each case, it can be observed that the total momentum before and after the collision in each case is constant as external forces are absent and internal forces are conservative in nature.

so initial momentum = final momentum

$p_i = p_f$

For an elastic collision, the total kinetic energy of a system remains conserved before and after a collision while for an inelastic collision kinetic energy is not conserved as some energy is lost due to work done by the internal forces.

Given the mass of ball bearings m, we can write:

The total kinetic energy of the system before the collision is given by:

$K.E. = \frac{1}{2}mV^2 + \frac{1}{2}(2m)0$

$K.E. = \frac{1}{2}mV^2$

Now,

In Case (a)

The total kinetic energy of the system after the collision is given by:

$K.E. = \frac{1}{2}m \times 0 + \frac{1}{2}(2m)\left(\frac{V}{2}\right)^2$

$K.E. = \frac{1}{4}mV^2$

Hence, in case (a) the kinetic energy of the system is not conserved.

In Case (b)

The total kinetic energy of the system after the collision is given by:

$K.E. = \frac{1}{2}(2m) \times 0 + \frac{1}{2}mV^2$

$K.E. = \frac{1}{2}mV^2$

Hence, in case (b) the kinetic energy of the system is conserved.

In Case (c)

The total kinetic energy of the system after the collision is given by:

$K.E. = \frac{1}{2}(3m)\left(\frac{V}{3}\right)^2$

$K.E. = \frac{1}{6}mV^2$

Hence, the total kinetic energy of the system is not conserved in case(b) & case(c) in the process of collision.

74. Initial kinetic energy,

$K = \frac{1}{2} mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$

$\therefore$ Initial momentum, $p = \sqrt{2mK}$

Increase in kinetic energy = 300% of K =3 K

Final kinetic energy,

K' = K + 3K = 4K

Final momentum,

$p' = \sqrt{2mK'} = \sqrt{2m \times 4K} = 2\sqrt{2mK}$ = 2p

% Increase in momentum

$= \frac{p^{-p}}{p} \times 100 = \frac{2p-p}{p} \times 100 = 100\%$.

75. Here

$m_1 = 9000$ kg, $u_1 = 36$ kmh^{-1} = 10 ms^{-1}

$m_2 = 9000$ kg, $u_2 = 0, v_1 = v_2 = v = ?$

By conservation of momentum,

$m_1u_1 + m_2u_2 = (m_1 + m_1)v$

$9000 \times 10 + 0 = (9000+ 9000)v$

$v = \frac{90000}{18000} = 5$ ms^{-1}

Total K.E. before collision

$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$= \frac{1}{2} \times 9000 \times 10 \times 10 + 0 = 450000$ J

Total K.E. after collision = $\frac{1}{2}(m_1 + m_2) v^2$

$= \frac{1}{2} \times 2 \times 9000 \times 5^2 = 225000$ J.

Thus total K.E. after collision < Total K.E. before the collision.

Hence the collision is inelastic.