VECTOR ALGEBRA

Class 11 - Physics

Section A

1. Which of the following physical quantities is a scalar? **[1]**

   a) average velocity
   b) linear momentum
   c) current
   d) relative velocity

2. Which of the following is vector quantity? **[1]**

   a) Charge
   b) Temperature
   c) Impulse
   d) Coefficient of friction

3. Acceleration due to gravity: **[1]**

   a) a scalar
   b) polar vector
   c) a vector
   d) a non scalar

4. If $A_x, A_y \; and \; A_z$ are x, y and z components of a vector then its magnitude is **[1]**

   a) $\sqrt{A_x^2 + 2A_y^2 + A_z^2}$
   b) $\sqrt{A_x^2 + A_y^2 + A_z^2}$
   c) $\sqrt{3A_x^2 + A_y^2 + A_z^2}$
   d) $\sqrt{A_x^2 + A_y^2 + A_z^3}$

5. Angular momentum is **[1]**

   a) Position Vector
   b) axial vector
   c) polar vector
   d) scalar

6. Multiplying a vector $\vec{v}$ by a positive real number λ **[1]**

   a) gives a vector $\overrightarrow{v'} = \lambda\vec{v}$ in a direction opposite to $\vec{v}$
   b) gives a scalar that is λ times the polar angle of $\vec{v}$
   c) gives a scalar that is λ times the magnitude of $\vec{v}$
   d) gives a vector $\overrightarrow{v'} = \lambda\vec{v}$ in the same direction as $\vec{v}$

7. Multiplying a vector $\vec{v}$ by a negative real number λ **[1]**

   a) gives a vector $\overrightarrow{v'} = \lambda\vec{v}$ in a direction opposite to $\vec{v}$
   b) gives a vector $\overrightarrow{v'} = \lambda\vec{v}$ in the same direction as $\vec{v}$
   c) gives a scalar that is λ times the magnitude of $\vec{v}$
   d) gives a scalar that is λ times the polar angle of $\vec{v}$

8. An arbitrary vector $\vec{A}$ in a plane can be expressed in terms of its x and y components by the equation **[1]**

a) $\vec{A} = A_x\hat{i} - A_y\hat{j}$ b) $\vec{A} = A_x + A_y$

c) $\vec{A} = A_x\hat{i} + A_y\hat{j}$ d) $\vec{A} = A_x - A_y$

9. A unit vector is a vector having a magnitude of 1 and points in **[1]**

a) any chosen direction b) x-direction

c) y-direction d) z-direction

10. The unit vectors along the three co-ordinate axes are related as: **[1]**

a) $\hat{i} = \hat{j} = \hat{k}$ = 1 b) $\hat{i} = -\hat{j} = \hat{k}$ = 1

c) $\hat{i} = \hat{j} = \hat{k}$ = 0 d) $\hat{i} > \hat{j} > \hat{k} > 1$

11. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$? **[1]**

a) $6\hat{i} - 2\hat{j} + 8\hat{k}$ b) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ d) $-6\hat{i} - 2\hat{j} + 3\hat{k}$

12. When $\vec{A} \cdot \vec{B} = -|\vec{A}||\vec{B}|$, then **[1]**

a) $\vec{A}$ and $\vec{B}$ act in the same direction b) $\vec{A}$ and $\vec{B}$ can act in any direction

c) $\vec{A}$ and $\vec{B}$ act in the opposite direction d) $\vec{A}$ and $\vec{B}$ act in the same direction

13. $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$, the value of θ is **[1]**

a) 30° b) 60°

c) 90° d) 45°

14. Angle between two vectors of magnitudes 12 and 18 units, when their resultant is 24 units, is **[1]**

a) 63°51' b) 82°31'

c) 75°52' d) 89°16'

15. If $\vec{P} + \vec{Q} = \vec{R}$ and $|\vec{P}| = |\vec{Q}| = |\vec{R}|$, then angle between $\vec{P}$ and $\vec{Q}$ is **[1]**

a) 30^o b) 60^o

c) 90^o d) 120^o

16. $\vec{A} = 4\hat{i} + 4\hat{j} - 4\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$, then angle between vectors $\vec{A}$ and $\vec{B}$ is **[1]**

a) 90° b) 180°

c) 0° d) 45°

17. If $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be **[1]**

a) 30^o b) 90^o

c) 60^o d) 45^o

18. The direction of $\vec{A}$ is vertically upward and $\vec{B}$ direction B is in north direction. The direction of $\vec{A} \times \vec{B}$ will be **[1]**

a) vertically downward b) at 45° upward in north

c) western direction d) eastern direction

19. The angle between the two vectors **[1]**

$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be

a) zero

b) 180°

c) 90°

d) 45°

20. The scalar product of two vectors A and B is **[1]**

a) a tensor

b) vector

c) a scalar

d) a complex number

21. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B} \times \vec{A}) \cdot \vec{A}$ is equal to **[1]**

a) $BA^2 \cos \theta$

b) $BA^2 \sin\theta\cos\theta$

c) $BA^2 \sin\theta$

d) zero

22. In a clock-wise system, **[1]**

a) $\hat{j} \times \hat{j} = \hat{i}$

b) $\hat{i} \times \hat{k}$ = 0

c) $\hat{k} \times \hat{i}$ = 1

d) $\hat{j} \times \hat{k} = \hat{i}$

23. The scalar product of vectors is **[1]**

a) non commutative and not distributive

b) non commutative and distributive

c) commutative and not distributive

d) commutative and distributive

24. The scalar product of two vectors A and B in terms of the magnitudes and angle θ is **[1]**

a) $|\mathbf{A}||\mathbf{B}|\cos\theta$

b) $|\mathbf{A}||\mathbf{B}|\tan\theta$

c) $|\mathbf{A}||\mathbf{B}|\sin\theta$

d) $|\mathbf{A}||\mathbf{B}|\cot\theta$

25. If $\vec{A} = \vec{B} + \vec{C}$ and the values of$\vec{A}$, $\vec{B}$ and $\vec{C}$ 13, 12 and 5 respectively, then the angle between $\vec{A}$ and $\vec{B}$ will be **[1]**

a) $\frac{\pi}{2}$

b) $\cos^{-1}(\frac{5}{13})$

c) $\cos^{-1}(\frac{13}{12})$

d) $\cos^{-1}(\frac{5}{12})$

26. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is **[1]**

a) 0°

b) 45°

c) 90°

d) 180°

27. To find the sum of vectors $\vec{A}$ and $\vec{B}$, we place vector $\vec{B}$ so that its **[1]**

a) tail is at the tail of the vector $\vec{A}$

b) direction is the same as that of vector $\vec{A}$

c) tail is at the head of the vector $\vec{A}$

d) head is at the head of the vector $\vec{A}$

28. Two vectors are given by $\vec{A} = (3\hat{i} + \hat{j} + 3\hat{k})$ and $\vec{B} = (3\hat{i} + 5\hat{j} - 2\hat{k})$. Find the third vector $\vec{C}$ if $\vec{A} + 3\vec{B} - \vec{C}$ = 0 **[1]**

a) $(12\hat{i}$ + $16\hat{j}$ - $3\hat{k})$

b) $(12\hat{i}$ + $14\hat{j}$ +12 $\hat{k})$

c) $(13\hat{i}$ + $17\hat{j}$ + $12\hat{k})$

d) $(15\hat{i}$ + $13\hat{j}$ + $4\hat{k})$

29. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces **[1]**

a) cannot be predicted

b) are not equal to each other in magnitude

c) are equal to each other

d) are equal to each other in magnitude

30. It is found that |A + B| = |A|.This necessarily implies, **[1]**

a) B = 0

b) A.B ≤ 0

c) A, B are perpendicular

d) A, B are antiparallel

31. We can define the difference of two vectors A and B as the sum of two vectors A and B' such that B' is equal to B multiplied by **[1]**

a) -1

b) -2

c) 0

d) 1

32. A cyclist moves in such a way that he takes 60° turn after every 100 metres. What is the displacement when he takes seventh turn? **[1]**

a) 200 m

b) 100 m

c) $\frac{100}{\sqrt{3}}m$

d) $100\sqrt{3}$ m

33. A displacement vector is a **[1]**

a) scalar

b) change in position

c) distance without direction

d) velocity

34. The position of a particle is given by $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 4.0\hat{k}$. Find the magnitude and direction of the velocity of the particle at t = 2.0 s. **[1]**

a) 8.84 m s^{-1}, 75° with x-axis

b) 6.54 m s^{-1}, 74° with x-axis

c) 7.54 m s^{-1}, 72° with x-axis

d) 8.54 m s^{-1}, 70° with x-axis

35. A particle is moving such that its position coordinates (x, y) are **[1]**
(2 m, 3 m) at time t =0,
(6 m, 7 m) at time t = 2s, and
(13 m, 14 m) at time t = 5 s
Average velocity vector $(\vec{v}_{av})$ from t = 0 to t = 5s is

a) $\frac{7}{3}(\hat{i} + \hat{j})$

b) $\frac{11}{5}(\hat{i} + \hat{j})$

c) $\frac{1}{5}(13\hat{i} + 14\hat{j})$

d) $2(\hat{i} + \hat{j})$

36. The magnitude of displacement of a particle is **[1]**

a) is equal to the path length of the particle between two points.

b) is less than the path length of the particle between two points.

c) is more than the path length of the particle between two points.

d) is either less or equal to the path length of the particle between two points.

37. A bird flies from (-3m, 4 m, -3 m) to (7 m, -2 m, -3 m) in the xyz coordinates. The bird's displacement in unit vectors is given by **[1]**

a) $(10\hat{i} + 6\hat{j} - 6\hat{k})$

b) $(4\hat{i} + 2\hat{j} - 6\hat{k})$

c) $(4\hat{i} - 2\hat{j})$

d) $(10\hat{i} - 6\hat{j})$

38. Does a scalar quantity depend on the frame of reference chosen? **[1]**

39. Is finite rotation a vector? **[1]**

40. If $\vec{A}, \vec{B}$ and $\vec{C}$ are mutually perpendicular vectors, then what is the value of $\vec{A} \cdot (\vec{B} + \vec{C})$? **[1]**

41. For any two vectors $\vec{A}$, $\vec{B}$, prove that$(\vec{A} \times \vec{B})^2 = A^2B^2 - (\vec{A} \cdot \vec{B})^2$ **[1]**

42. When vectors are added or substrated, their resultant is a vector. Is it also true in the case of multiplication of two vectors? **[1]**

43. Under what condition is the scalar product of two non-zero vectors zero? **[1]**

Section B

44. What are vector quantities? Give examples too. **[2]**

45. We can order events in time and there is no sense of time, distinguishing past, present and future. Is time a vector? **[2]**

46. Is $|\vec{a} + \vec{b}|$ greater than or less than $|\vec{a}| + |\vec{b}|$? Given reason. **[2]**

47. A vector $\vec{a}$ is turned through a small angle do without a change in its length. What are $|\Delta\vec{a}|$ and Δa? **[2]**

48. Establish the $|a - b| \geq ||a| - |b||$ vector inequalities geometrically or otherwise.When does this equality sign apply? **[2]**

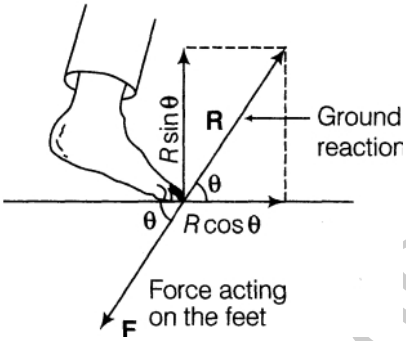
49. Suppose you have two forces $\vec{F}$ and $\vec{F}$. How would you combine them in order to have resultant force of magnitudes? **[2]**

    i. Zero

    ii. $2\vec{F}$

    iii. $\vec{F}$

50. Can the walk of a man be an example of resolution of forces? **[2]**



51. Establish the **|a + b| ≤ |a| + |b|** vector inequalities geometrically or otherwise. When does this equality sign apply? **[2]**

52. What is a unit vector? How would you obtain unit vector of a given vector? **[2]**

53. The sum and difference of two vectors $\vec{A}$ and $\vec{B}$ are $\vec{A} + \vec{B} = 2\hat{i} + 6\hat{j} + \hat{k}$ and $\vec{A} - \vec{B} = 4\hat{i} + 2\hat{j} - 11\hat{k}$. Find the magnitude of each vector and their scalar product $\vec{A} \cdot \vec{B}$. **[2]**

54. Find the area of a parallelogram formed by vectors $\vec{A} = 3\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} + 7\hat{j}$ **[2]**

55. If vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ have magnetic 5, 12 and 13 units and $\vec{P} + \vec{Q} = \vec{R}$, find the angle between $\vec{Q}$ and $\vec{R}$. **[2]**

56. Prove that the vectors $\vec{A} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 12\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel to each other. **[2]**

57. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$ and its magnitude is half that of $\vec{Q}$. What is the angle between $\vec{P}$ and $\vec{Q}$? **[2]**

58. A force $\vec{F} = 5\hat{i} + 4\hat{j}$ newton displaces a body through $\vec{S} = 3\hat{i} + 4\hat{k}$ metre in 3s. Find the power. **[2]**

59. The dot product of two vectors vanishes when vectors are orthogonal and has maximum value when vectors are parallel to each other. Explain. **[2]**

60. Explain the property of two vectors $\vec{A}$ and $\vec{B}$ if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. **[2]**

61. ABCDE is a pentagon. Prove that **[2]**
    $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \vec{0}$

62. Show that the vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right angle triangle **[2]**

63. State and prove the commutative property of vector addition. **[2]**

64. At what angle do the two forces (P + Q) and (P - Q) act so that the resultant is $\sqrt{3P^2 + Q^2}$? **[2]**

65. Do $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ lie in the same plane. Give reason. **[2]**

66. Is $|\vec{a} - \vec{b}|$ greater than or less than $|\vec{a}| + |\vec{b}|$? **[2]**

67. A body is simultaneously given two velocities, one 30 ms^{-1} due east and other 40 ms^{-1} due north. Find the resultant velocity. **[2]**

68. ABCD is a parallelogram and $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are its diagonals. Prove that: **[2]**

i. $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$ and

ii. $\overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$

69. If the resultant of the vectors $3\hat{i} + 4\hat{j} + 5\hat{k}$ and $5\hat{i} + 3\hat{j} + 4\hat{k}$ makes an angle θ with x-axis, then find $\cos\theta$. **[2]**

70. If $\vec{A} = \vec{B} - \vec{C}$, then determine the angle between $\vec{A}$ and $\vec{B}$. **[2]**

Section C

71. In any ΔABC, prove that **[3]**

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

72. Can a flight of a bird, an example of composition of vectors. Why? **[3]**

73. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: **[3]**

i. adding any two scalars,

ii. adding a scalar to a vector of the same dimensions,

iii. multiplying any vector by any scalar,

iv. multiplying any two scalars,

v. adding any two vectors,

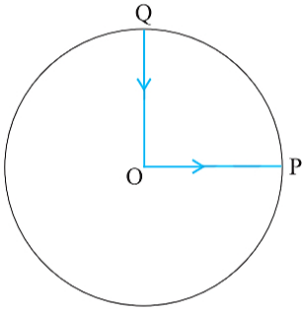
vi. adding a component of a vector to the same vector.

74. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the **[3]**

i. net displacement,

ii. average velocity, and

iii. average speed of the cyclist?



Section D

75. On an open ground, a motorist follows a track that turns to his left by an angle of 60^o after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case. **[5]**