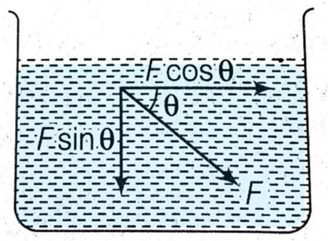
Mechanical properties of fluids

Most important question answers

Question (1): Prove that a fluid always exerts a force

Consider a liquid contained in a vessel in equilibrium state of rest as shown. Suppose the liquid exerts a force F on the bottom surface in an inclined direction.



The force F has two rectangular components:

i. Tangential component Fcosθ

ii. Normal component Fsinθ

Since a liquid cannot resist any tangential force, so liquid should begin to flow along horizontal. But the liquid is at rest, the force along horizontal must be zero.

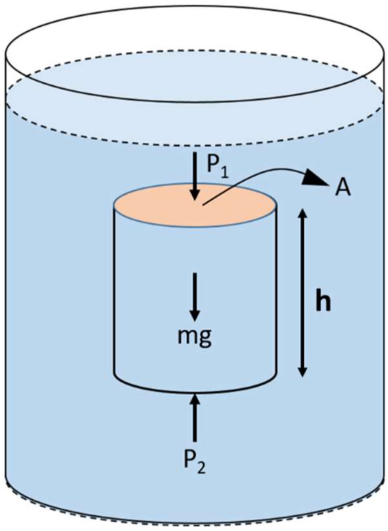
$\therefore F\cos\theta = 0$

As $F \neq 0$, so $\cos\theta = 0$ or $\theta = 90^0$

Question (2): Derive an expression for pressure at a depth h in a fluid.

Imagine a cylindrical element of the liquid of cross-sectional area A and height h. Let P_1 and P_2 be the liquid pressures at its top point 1 and bottom point 2 respectively.

Various force acting on it in the vertical direction are:



- Force due to the liquid pressure at the top,

  $F_1 = P_1A$, acting downwards

- Force due to the liquid pressure at the bottom,

  $F_2 = P_2A$, acting upwards

- Weight of the liquid cylinder acting downwards,

$W = \text{mass} \times g = \text{Volume} \times \text{density} \times g$

$\Rightarrow W = V\rho g$

As the liquid cylinder is in equilibrium,

$\therefore F_1 + W = F_2$

$\Rightarrow F_2 - F_1 = W$

$\Rightarrow P_2A - P_1A = Ah\rho g$

$\Rightarrow \boxed{P_2 - P_1 = h\rho g}$

If we shift the point 1 to the liquid surface, which is open to the atmosphere, then we can replace P_1 by atmospheric pressure P_a and P_2 by P in the above equation and we get

$P - P_a = h\rho g$

$\boxed{P = P_a + h\rho g}$

Pressure P is called total pressure and pressure $h\rho g$ at depth h is called a **gauge pressure** at that point.

Question (3): Define surface tension. Write its SI unit

Surface tension is a property of liquids where the molecules at the surface are attracted to each other more strongly than those in the bulk of the liquid, causing the surface to behave like an elastic sheet. Its SI unit is Newtons per meter (N/m).

Question (4): What are cohesive and adhesive forces? Explain giving examples.

1. **Cohesive Forces**: These are the forces of attraction that occur between similar molecules. Think of them as a kind of "team spirit" that molecules of the same substance have, pulling each other together. A common example is water. Have you noticed how water forms drops? That's because the water molecules are attracted to each other, holding the drop together. This is due to cohesive forces.
2. **Adhesive Forces**: These forces happen between different kinds of molecules. Imagine it as a friendship between two different types of molecules. They like to stick to each other. A daily life example is when you spill water on a table, and the water spreads out and sticks to the table. The water molecules (liquid) are being attracted to the table molecules (solid), demonstrating adhesive forces.

Question (5): What is angle of contact?

When a liquid comes into contact with another medium, its surface often curves near the point of contact.

The "angle of contact" is defined as the angle formed between the tangent to the liquid surface at the contact point and the solid surface inside the liquid.

This angle is represented by the symbol θ.

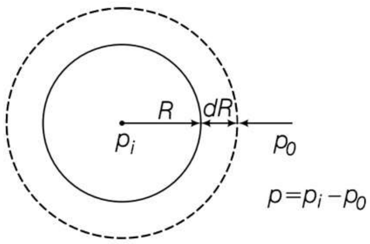
Several factors influence the value of the angle of contact:

1. The characteristics of the solid and liquid that are in contact.
2. The level of cleanliness of the contacting surface.
3. The nature of the medium present above the liquid's free surface.
4. The temperature of the liquid.

Question (6): Derive an expression for excess pressure inside a liquid drop and soap bubble.

Excess pressure inside a liquid drop

Suppose there is a spherical liquid drop with a radius R and let S be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop compared to the outside. This excess pressure acts outwardly. When this pressure causes the radius to increase from R to R + dR, the additional surface energy associated with this change can be determined.



Excess pressure inside a liquid drop

Excess pressure inside the drop, $p = p_i - p_o$

Where,

p_i = total pressure inside the liquid drop

p_o = atmospheric pressure

initial surface area of the liquid

$= 4\pi R^2$

final area of the liquid drop

$$= 4\pi(R + dR)^2$$
$$= 4\pi(R^2 + 2RdR + dR^2)$$
$$= 4\pi R^2 + 8\pi RdR \qquad [dR^2 \text{ is very so it is neglected}]$$

Increase in surface area of liquid drop

$= 4\pi R^2 + 8\pi RdR - 4\pi R^2$

$= 8\pi RdR$

External work done in increasing area of the drop

= increase in surface energy

= increase in surface area $\times$ surface tension

= $(8\pi RdR) \times S$ ....(i)

But work done

= excess pressure $\times$ Area $\times$ Surface tension

= $p \times 4\pi R^2 \times dR$ ....(ii)

From eq. (i) and (ii), we get

$p \times 4\pi R^2 \times dR = 8\pi RdRS$

$$\boxed{\text{Excess pressure} = p = \frac{2S}{R}}$$

So pressure difference in a drop bubble

$$\boxed{p_i - p_o = \frac{2S}{R}}$$

Excess pressure inside a soap bubble

From the above case, increase in surface area = $8\pi RdR$

But a soap bubble has two free surfaces

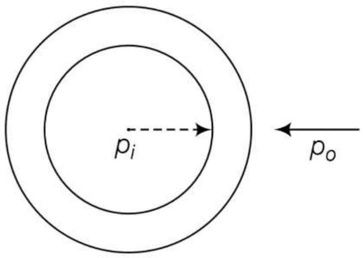
So, effective increase in surface area of the soap bubble $= 2 \times 8\pi RdR = 16\pi RdR$ ......(i)

External work done in increasing the surface area of the soap bubble

= increase in surface energy

= increase in surface area $\times$ surface tension

$= 16\pi RdRS$



Excess pressure inside a soap bubble

But, work done = force $\times$ change in radius

Where, force = Excess pressure $\times$ area = $(p \times 4\pi r^2)$

So, work done = $p \times 4\pi R^2 \times dR$ .....(ii)

From (i) and (ii), we get

$p \times 4\pi R^2 \times dR = 16\pi R dRS$

$$\boxed{p = \frac{4S}{R}}$$

Pressure difference inside a soap bubble

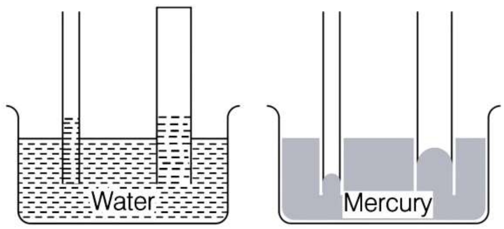
$$\boxed{p_i - p_o = \frac{4S}{R}}$$

Excess pressure inside an air bubble in a liquid is similar to a liquid drop in air, it has only one free spherical surface. Hence, excess pressure is given by

$$\boxed{p = \frac{2S}{R}}$$

Question (7): What is capillarity? Derive Ascent formula.

The word 'capilla' comes from Latin, meaning 'hair.' A tube with a very thin bore, resembling a hair, is known as a capillary tube. When this tube is immersed in a liquid like water, the water level inside the tube rises. In contrast, with a liquid like mercury, the level inside the tube drops when it's dipped. This behaviour of liquids – either rising or falling in a capillary tube – is termed capillarity.



Capillarity

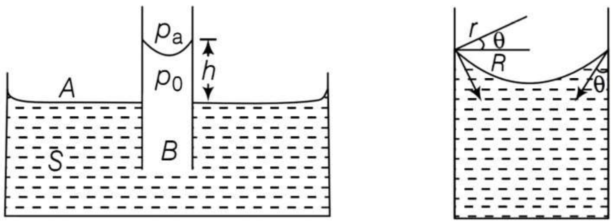
An application of the pressure difference across a curved surface is when water rises up in a narrow tube (capillary) despite the force of gravity. Consider a capillary tube with a radius R that is inserted into a vessel containing water. The surface of water in the capillary becomes concave. This indicates that there must be a pressure difference between the two sides of the meniscus.

The pressure difference

$$p_a - p_o = \frac{2S}{r} = \frac{2S}{R\sec\theta}$$

$$= \left(\frac{2S}{R}\right)\cos\theta \qquad \text{.........(i)}$$

where r is the radius of curvature of the concave meniscus.



Capillary rise

Now, consider two points A and B. According to Pascal's law, they must be at the same pressure.

$$p_o + h\rho g = p_A = p_B = p_a$$

$$p_a = p_o + h\rho g$$

$$p_a - p_o = \rho g h \qquad \text{........(ii)}$$

$$(p_a = \text{atmospheric pressure})$$

From eq. (i) and (ii), we get

$$\rho g h = \frac{2S}{R}\cos\theta$$

$$h = \frac{2S\cos\theta}{R\rho g}$$

The liquids which met the glass surfaces, e.g. water, rises in the capillary and the liquids which do not wet the glass surface falls in the capillary.

Question (8): How does surface tension vary with temperature? Explain giving examples.

The surface tension of a liquid goes down when the temperature goes up, and it increases when the temperature drops. Each liquid has a special temperature called the 'Critical temperature,' at which its surface tension becomes zero.

For small temperature differences, surface tension decreases almost linearly as $S_t = S_o(1-\alpha t)$ where:

- S_t: Surface tension at $t^\circ C$,
- S_o: Surface tension at $0^\circ C$, and
- α : the temperature coefficient of surface tension.

Due to this effect:

- Hot soup has a better flavor than cold soup because it spreads more across the tongue.
- In winter, machinery parts often get stuck because the surface tension is higher.

Question (9): What is viscosity?

Viscosity is a measure of a fluid's resistance to flow. It describes how easily a fluid deforms or flows in response to an applied force.

In simpler terms, viscosity is the thickness or "stickiness" of a fluid. High-viscosity fluids, like honey or molasses, flow slowly because they offer a lot of resistance to deformation. In contrast, low-viscosity fluids, like water or gasoline, flow more easily because they offer less resistance.

Viscosity depends on factors such as temperature, pressure, and the composition of the fluid. For example, fluids generally become less viscous as temperature increases.

The SI unit of viscosity is the Pascal-second (Pa·s) or the Poise (P).

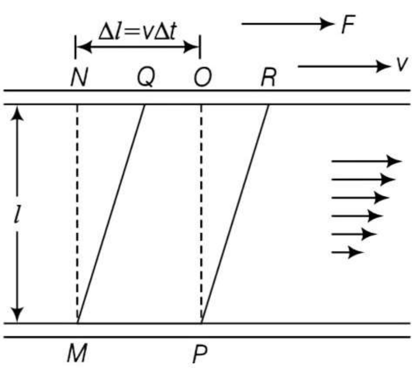
Question (10): What is coefficient of viscosity? Write its SI units and dimensions.

Consider the flow of liquid as shown in figure. A portion of liquid which at some instant having the shape MNOP after the short interval of time (say Δt) the fluid is deformed and take the shape as MQRP since, the fluid has undergone the shear strain, stress in the solid is the force per unit area but in case of fluid, it depends on the rate of change of strain or strain rate.

Strain $\frac{\Delta \ell}{\ell}$ and the rate of change of strain is $\frac{\Delta \ell}{\ell \Delta t} = \frac{v}{\ell}$.

Hence, the coefficient of viscosity is defined as the ratio of shearing stress to the strain rate

$$\eta = \frac{F/A}{v/\ell} = \frac{F\ell}{vA}$$



The layer of fluid is deformed under some stress

$$\text{coefficient of viscosity, } \eta = \frac{F}{A} \cdot \frac{\ell}{v}$$

or it can be written as, $\eta = \dfrac{F/A}{\frac{dv}{dx}}$

Dimensions of η

$$\eta = \frac{[MLT^{-2}]}{[L^2LT^{-1}]}[L] = [ML^{-1}T^{-1}]$$

The coefficient of viscosity is a scalar quantity, indicating that it has magnitude but no directional property.

Units of coefficient of viscosity

- In cgs system, the unit of η is dyne.s/cm^2 and it is called poise

$$1 \text{ poise} = \frac{1 \text{ dyne}}{1 \text{ cm}^2} \times \frac{1 \text{ cm}}{1 \text{ cm/s}}$$

$$1 \text{ poise} = 1 \text{ dyne.s/cm}^2$$

The coefficient of viscosity for a liquid is defined as 1 poise when a tangential force of 1 dyne per square centimetre is needed to maintain a relative velocity of 1 cm/s between two liquid layers that are 1 cm apart.

- The SI unit of η is Ns/m^2 and it is called decapoise or poiseuille.

$$1 \text{ poiseuille} = \frac{1 \text{N}}{1 \text{ m}^2} \cdot \frac{1 \text{ m}}{1 \text{ m/s}} = 1 \text{Ns/m}^2$$

The coefficient of viscosity of a liquid is defined as 1 poiseuille (or 1 decapoise) when a tangential force of 1 Newton per square meter (Nm²) is required to maintain a relative velocity of 1 meter per second (m/s) between two layers of the liquid that are 1 meter apart.

Relation between Poiseuille and Poise

1 poiseuille or 1 decapoise is equivalent to 10 poise. This conversion factor helps in interchanging units between the CGS system (poise) and the SI system (poiseuille).

Question (11): What is Stokes's law?

Concept: When a small spherical object moves through a viscous fluid, it experiences a drag force due to the viscosity of the fluid. This drag force acts in the opposite direction to the object's motion.

Formula: The drag force *F* exerted on the spherical object is given by Stoke's law as:

$$F = 6\pi\eta rv$$

where

- η (Eta) is the coefficient of viscosity of the fluid.
- *r* is the radius of the spherical object.
- *v* is the velocity of the object relative to the fluid.

Let $F = k\eta^a v^b r^c$ .....(i)

As $[F] = [MLT^{-2}], [\eta] = [ML^{-1}T^{-1}]$

$[v] = [LT^{-1}], [r] = [L]$

So, $[MLT^{-2}] = [ML^{-1}T^{-1}][LT^{-1}]^b [L]^c$

$[MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-b}]$

comparing powers, we get

$a = 1$ (ii)

$-a + b + c = 1$ (iii)

$-a - b = 2$ (iv)

$a + b = 2$ (v)

From Eqs. (iii) and (v), we get

$c = 1, b = 1$

Substituting these values in Eq. (i), we get $F = 6\pi\eta r v$

where, the value of k was found to be 6π experimentally.

Question (12): What is terminal velocity? Derive an expression for terminal velocity acquired by a body while falling freely in a viscous medium.

The maximum constant velocity acquired by a body while falling through a viscous fluid is called its terminal velocity.

Expression for terminal velocity. Consider a spherical body of radius r falling through a viscous liquid of density of the body.

As the body falls, the various forces acting on the body are:

1. Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

2. Upward thrust equal to the weight of the liquid displaced.

$$F_B = \frac{4}{3}\pi r^3 \sigma g$$

3. Force of viscosity acting in the upward direction. According to Stoke's Law,

$$F_v = 6\pi\eta r v$$

When the body attains terminal velocity v ,

$$F_V + F_B = W$$

$$\Rightarrow \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow 6\pi\eta r v_t = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

Or $$v_t = \frac{2r^2(\rho - \sigma)}{9\eta} g$$

This is the expression for terminal velocity.

Question (13): What is Reynold's number? Write its value of the various types of flows.

Reynold's Number, denoted as R_e, is a dimensionless parameter that helps predict the flow pattern of a fluid in a pipe or another conduit. It's given by the formula:

$$R_e = \frac{\rho v D}{\eta}$$

where:

- ρ is the density of the liquid.
- v is the velocity of the liquid.
- D is the diameter of the pipe.
- η is the coefficient of viscosity of the liquid.

Based on the value of *Re*, the flow can be categorized as:

1. Laminar or Streamline Flow: R_e between 0 and 2000. The flow is smooth and orderly.
2. Transitional Flow: R_e between 2000 and 3000. The flow can switch between laminar and turbulent.
3. Turbulent Flow: R_e greater than 3000. The flow is chaotic and irregular.

The exact value at which turbulent sets in a fluid is called critical Reynold's number. In another form, R_e, can also be written as

$$R_e = \frac{\rho v D}{\eta} = \frac{\rho v^2}{\left(\eta \frac{v}{D}\right)} = \frac{\rho A v^2}{\eta \frac{Av}{D}} = \frac{\text{inertial force}}{\text{force of viscosity}}$$

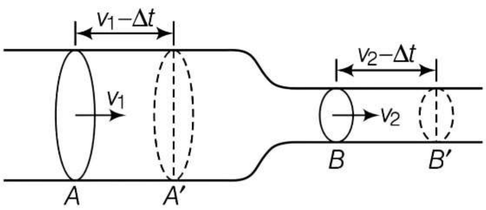
Hence, Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

Question (14): Derive equation of continuity.

This principle states that for a non-viscous and incompressible fluid flowing in streamline motion through a pipe with varying cross-sections, the product of the cross-sectional area and the normal fluid velocity (denoted as av) remains constant throughout the flow.

Consider a non-viscous and incompressible liquid flowing through a tube AB with a varying cross-section.

Let a_1 be the area of cross-section and v_1 the velocity at section A of the tube. Correspondingly, let a_2 and v_2 be the area of cross-section and the velocity at section B, respectively. The density of the flowing fluid is represented as ρ



Fluid flow through pipe *AB*

Since the fluid is incompressible, its density (ρ) is constant.

Recall that mass is equal to volume multiplied by density. Thus, for a constant density (ρ), the mass of the fluid can be expressed in terms of its volume.

Let's consider the mass of fluid passing through sections A and B of the pipe:

1. **At Section A:**

The cross-sectional area is a_1 and the velocity is v_1. The volume of fluid passing through section A in a small time interval Δt is $a_1 \times v_1 \times \Delta t$. Hence, the mass of fluid entering through section A in time Δt (denoted as m_1) is given by:

$$m_1 = a_1 \times v_1 \times \Delta t \times \rho$$

2. **At Section B:**

Similarly, for section B, with a cross-sectional area of a_2 and a velocity v_2, the mass of fluid leaving through section B in time Δt (denoted as m_2) is:

$$m_2 = a_2 \times v_2 \times \Delta t \times \rho$$

According to the principle of conservation of mass, the mass entering the pipe must equal the mass leaving it, so $m_1 = m_2$. Therefore, we have:

$$a_1 \times v_1 \times \Delta t \times \rho = a_2 \times v_2 \times \Delta t \times \rho$$

Since ρ is constant and the same for both sections, and Δt is also the same for both intervals, we can simplify this equation to:

$$\boxed{a_1 \times v_1 = a_2 \times v_2}$$

Or more generally,

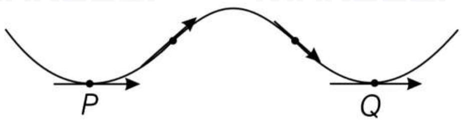
$$\boxed{av = \text{constant}}$$

This relationship is known as the Equation of Continuity.

Question (15): What are three types of flow, name and explain them.

(1) Streamline Flow

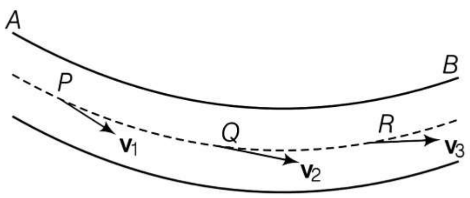
The streamline flow of a liquid is a type of flow in which each particle of the liquid passing through a particular point follows the same path and moves at the same velocity as the preceding particle passing through that point. It can also be defined as a curve where, at any point, the tangent to the curve aligns with the fluid velocity at that point.



Trajectory of a fluid particle

Properties of Streamline Flow

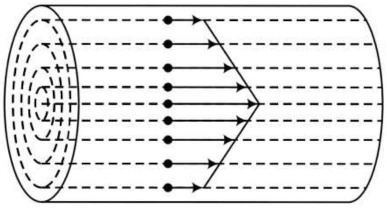
1. In streamline flow, no two streamlines can intersect each other. If they did, the liquid particles at the intersection would have to follow two different directions, disrupting the steady nature of the flow.
2. The density of streamlines in an area correlates with the liquid's velocity there: the more crowded the streamlines, the higher the velocity of the liquid particles in that area, and vice versa.



A region of streamline flow

(2) Laminar Flow

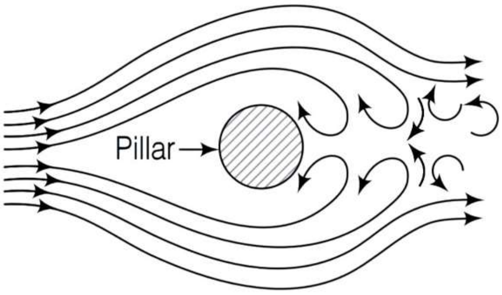
Laminar flow occurs when a liquid flows in smooth layers, each moving at a different speed, over a horizontal surface. In this kind of flow, the particles in one layer don't mix with those in another. It's usually a kind of streamline flow, where the fluid particles glide in orderly paths, staying in their layers without any crossover or mixing.



Laminar flow of liquid

(3) Turbulent Flow

Turbulent flow happens when the fluid moves quickly or when its path is sharply altered by surfaces it flows past. This type of flow is typical in rivers and canals. In turbulent flow, the speed of the fluid particles at a specific point is constantly changing, creating a chaotic and disorganized movement. This results in a disorderly flow with swirls and eddies, quite different from the smooth, layered movement seen in laminar flow.



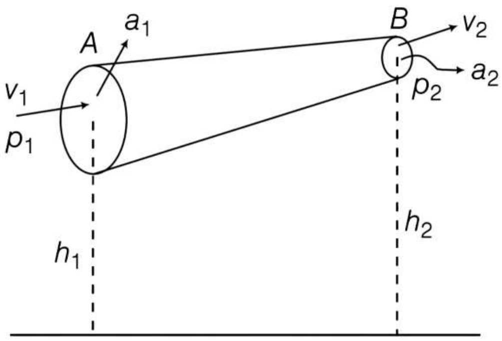
Turbulent flow of liquid

Question (16): State and prove Bernoulli's theorem.

Bernoulli's principle is fundamentally based on the law of conservation of energy, and it's applied to ideal fluids.

This principle states that for an incompressible, non-viscous fluid flowing in a streamlined and irrotational manner, the total energy per unit volume remains constant at every point along the flow.

This total energy is the sum of the fluid's pressure energy per unit volume, kinetic energy per unit volume, and potential energy per unit volume.



The flow of an ideal fluid in a pipe varying cross-section

Mathematically, Bernoulli's Theorem is expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

where, p represents for pressure energy per unit volume $\frac{1}{2}\rho v^2$ for kinetic energy per unit volume and ρgh for potential energy per unit volume and p is density of flowing fluid (ideal). The Swiss physicist Daniel Bernoulli developed this relationship in 1738.

Proof: Consider an ideal fluid having streamline flow through a pipe of varying area of cross-section as shown in figure.

Let P_1 and P_2 represent the pressures at points A and B, respectively. Let a_1 and a_2 be the areas of cross-section, h_1, h_2 the heights, and v_1, v_2 the velocities of flow at points A and B, respectively.

The force acting on the fluid at point A is given by $P_1 \times a_1$. The distance travelled by the fluid in one second at point A is its velocity v_1 . Therefore, the work done per second on the fluid at point A (denoted as W_1) is the product of the force and the distance travelled per second:

$$W_1 = P_1 \times a_1 \times v_1$$

Similarly, the work done per second by the fluid at point B (denoted as W_2) is:

$$W_2 = P_2 \times a_2 \times v_2$$

The net work done on the fluid by pressure energy (W) is the difference between the work done at points A and B:

$$W = P_1 \times a_1 \times v_1 - P_2 \times a_2 \times v_2$$

However, according to the Equation of Continuity, $a_1 v_1 = a_2 v_2 = \rho m$, where m is the mass flow rate and ρ is the density. Substituting this into the equation for net work, we get:

$$W = \left(\frac{P_1 m}{\rho}\right) - \left(\frac{P_2 m}{\rho}\right)$$

Total work done by the pressure energy on the fluid increases the kinetic energy and potential energy of the fluid, when it flows from A to B. Hence, Increase in potential energy of fluid

$$= \text{KE at B} - \text{KE at A}$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \qquad [\because a_1 > a_2 \therefore v_2 < v_1]$$

Similarly, total increase in potential energy = $mgh_2 - mgh_1$,

According to work energy theorem, work done on the fluid is equal to change in the energy of fluid.

i.e. work done by the pressure energy = total increase in energy

$$\frac{p_1 m}{\rho} - \frac{p_2 m}{\rho} = (mgh_2 - mgh_1) + \left(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2\right)$$

$$\left(\frac{p_1 - p_2}{\rho}\right) = (gh_2 - gh_1) + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

$$\frac{p_1}{\rho} + \frac{1}{2} v_1^2 + gh_1 = \frac{p_2}{\rho} + \frac{1}{2} v_2^2 + gh_2$$

$$\frac{p}{\rho} + gh + \frac{1}{2} v^2 = \text{constant}$$

Question (17): Explain the working of venturi meter.

A Venturimeter is a device used to measure the flow speed of an incompressible fluid through a pipe. It is also known as a flow meter or a Venturi tube.

Construction: The Venturimeter consists of a horizontal tube with a wider opening of cross-section a_1, and a narrow neck of cross-section a_2. These two regions of the horizontal tube are connected to a manometer, which contains a liquid of density σ.

The design of the Venturimeter allows it to utilize the principles of fluid dynamics to measure the flow speed accurately by observing changes in pressure at different cross-sectional areas of the tube.

Working. Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity,

$a_1 v_1 = a_2 v_2$ or

$$\frac{a_1}{a_2} = \frac{v_2}{v_1}$$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) \qquad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right]$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

$$P_1 - P_2 = h\sigma g$$

$$\therefore \quad h\sigma g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

$$\therefore \quad v_1 = \sqrt{\frac{2h\sigma g}{\rho} \times \frac{a_2^2}{a_1^2 - a_2^2}}$$

Volume of the liquid flowing out per second ,

$$\boxed{Q = a_1 v_1 = a_1 a_2\sqrt{\frac{2h\sigma g}{\rho(a_1^2 - a_2^2)}}}$$

Question (18): State and prove Torricelli's theorem.

According to Torricelli's Law, the velocity of efflux, i.e., the velocity with which the liquid flows out of an orifice (a narrow hole), is equal to the velocity that a freely falling body would acquire in falling through a vertical distance equal to the depth of the orifice below the free surface of the liquid.

Speed of Efflux: The term "efflux" refers to the outflow of a fluid, as depicted in the figure. Consider a tank containing a liquid of density *ρ*. Imagine a small hole on its side at a height y_1 from the bottom, and let y_2 be the height of the liquid surface from the bottom. Let P represent the air pressure above the liquid surface.

If A_1 and A_2 are the cross-sectional areas, and v_1 and v_2 are the velocities of the liquid at points 1 and 2, respectively, then from the equation of continuity, we get

$$A_1 v_1 = A_2 v_2 \text{ or } v_2 = \frac{A_1}{A_2} v_1$$

If $A_2 >>> A_1$, so the liquid may be taken at rest at the top, i.e $v_2 \approx 0$, or Applying the Bernoulli's theorem at points 1 and 2. The pressure $p_1 = p_a$ (atmospheric pressure)

We get

$$p_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p + \rho g y_2 \quad [v_2 \approx 0]$$

$$\text{or} \quad \frac{1}{2}\rho v_1^2 = \rho g(y_2 - y_1) + (p_2 - p_1)$$

$$y_2 - y_1 = h$$

$$\text{Hence,} \quad \frac{1}{2}\rho v_1^2 = \rho g h + (p - p_a)$$

Velocity of the liquid falling from orifice

$$v_1 = \sqrt{2gh + \frac{2(p - p_a)}{\rho}}$$

When the tank is open to the atmosphere

$$p = p_a$$

$$v_1 = \sqrt{2gh}$$

Thus, the velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called Torricelli's law. Distance at which the stream strikes the floor

$$x = 2\sqrt{hy_1}$$