Motion in a straight line

Most important questions

Question (1): Define the following: Speed, Velocity, Distance, Displacement, Uniform motion and uniform speed (non-uniform motion and non-uniform speed), acceleration.

Speed: Speed is the rate at which an object covers distance. It is a scalar quantity and is defined as the distance travelled per unit time. Mathematically, it is expressed as

$$\boxed{\text{speed} = \frac{\text{distance}}{\text{time}}}$$

Velocity: Velocity is the rate of change of displacement with respect to time. It is a vector quantity and includes both the magnitude (speed) and direction of motion. Mathematically, it is expressed as

$$\boxed{\text{velocity} = \frac{\text{displacement}}{\text{time}}}$$

Distance: Distance is the total length of the path travelled by an object. It is a scalar quantity and does not consider direction.

Displacement: Displacement is the change in position of an object. It is a vector quantity and is the shortest distance between the initial and final positions, including direction.

Uniform motion and uniform speed (non-uniform motion and non-uniform speed):

- Uniform motion: Uniform motion occurs when an object travels equal distances in equal intervals of time. In uniform motion, the speed remains constant over time.
- Uniform speed: Uniform speed is when an object covers equal distances in equal intervals of time, regardless of the direction. It doesn't necessarily mean that the object is moving in a straight line.
- Non-uniform motion: Non-uniform motion occurs when an object covers unequal distances in equal intervals of time or when the speed changes over time.
- Non-uniform speed: Non-uniform speed refers to the situation where the speed of an object changes over time, either increasing or decreasing.

Acceleration: Acceleration is the rate of change of velocity with respect to time. It is a vector quantity and can involve changes in speed, direction, or both. Mathematically, it is expressed as

$$\boxed{\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}}$$

If the velocity increases, the acceleration is positive; if it decreases, the acceleration is negative.

Question (2): Derive three equations of uniformly accelerated motion using graphical method.

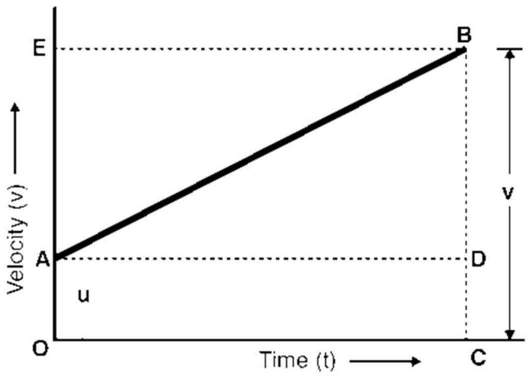
1st equation of motion

$a = \text{slope of } v - t \text{ graph}$

$$a = \frac{BD}{AD} = \frac{BC - DC}{AD}$$

$$\Rightarrow a = \frac{v - u}{t}$$

$$\Rightarrow \boxed{v = u + at}$$



2^{nd} equation of motion

$s = \text{area under } v - t \text{ graph}$

$s = \text{area of rectangle OACD + area of triangle ADB}$

$$s = OA \times OC + \frac{1}{2} AD \times DB$$

$$\Rightarrow s = OA \times OC + \frac{1}{2} AD \times (BC - DC)$$

$$\Rightarrow s = ut + \frac{1}{2} t(v - u)$$

$$\Rightarrow s = ut + \frac{1}{2} t \times at$$

$$\Rightarrow \boxed{s = ut + \frac{1}{2} at^2}$$

3^{rd} equation of motion

$s = \text{area under } v - t \text{ graph}$

$\Rightarrow s = \text{area of trapezium OABC}$

$$\Rightarrow s = \frac{1}{2}(\text{sum of parallel sides}) \times (\text{distance between them})$$

$$\Rightarrow s = \frac{1}{2}(OA + BC)(AD)$$

$$\Rightarrow s = \frac{1}{2}(u + v)(t)$$

$$\Rightarrow s = \frac{1}{2}(v + u)\left(\frac{v - u}{a}\right) \quad \text{[from eq(i)]}$$

$$\Rightarrow s = \frac{1}{2}\frac{(v^2 - u^2)}{a}$$

$$\Rightarrow \boxed{2as = v^2 - u^2}$$

Question (3): Derive three equations of uniformly accelerated motion using algebraic method.

Consider a body whose velocity charges from u to v in time t. Let acceleration be a, then

1^{st} equation of motion

By definition of acceleration, we have

$$a = \frac{v - u}{t}$$

$$\Rightarrow v - u = at$$

$$\Rightarrow \boxed{v = u + at}$$

2^{nd} equation of motion

$$\because v_{avg} = \frac{v+u}{2}$$

$$\therefore s = v_{avg} \times t$$

$$\Rightarrow s = \left(\frac{v+u}{2}\right) \times t \qquad \text{.......(i)}$$

$$\Rightarrow s = \left(\frac{u+at+u}{2}\right) \times t$$

$$\Rightarrow s = \left(\frac{2u+at}{2}\right) \times t$$

$$\Rightarrow s = \left(\frac{2u}{2} + \frac{at}{2}\right) \times t$$

$$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

3rd equation of motion

Putting the value of t from equation 1 in (i), we get

$$\Rightarrow s = \left(\frac{v+u}{2}\right) \times \left(\frac{v-u}{a}\right)$$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow \boxed{v^2 - u^2 = 2as}$$

Question (4): Derive three equations of uniformly accelerated motion using calculus method.

Consider a body moving with velocity u at t = 0 and velocity v at time t. Let the acceleration of the body be v and displacement during this time is s. Consider a small time interval dt during the motion of the body, let the velocity change in dt be dv and displacement be ds, then

First equation

$$a = \frac{dv}{dt}$$

$$dv = adt$$

integrating both sides, we get

$$\int_u^v dv = a\int_0^t dt$$

$$(v)_u^v = a(t)_0^t \Rightarrow v - u = at \Rightarrow \boxed{v = u + at}$$

Second equation

$$\text{As } s = v_{avg} \times t$$

$$\Rightarrow s = \left(\frac{v+u}{2}\right) \times t$$

$$\Rightarrow s = \left(\frac{u+at+u}{2}\right) \times t$$

$$\Rightarrow s = \left(\frac{2ut}{t} + \frac{at^2}{t}\right) \Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

Third equation

$$a = \frac{dv}{dt}$$

$$\Rightarrow a = \frac{dv}{ds} \times \frac{ds}{dt}$$

$$\Rightarrow a = \frac{dv}{ds} \times v$$

$$\Rightarrow a\,ds = v\,dv$$

$$\Rightarrow a\int_0^s ds = \int_u^v vdv$$

$$\Rightarrow a(s)_0^s = \left(\frac{v^2}{2}\right)_u^v$$

$$\Rightarrow a(s-0) = \frac{v^2}{2} - \frac{u^2}{2}$$

$$\Rightarrow \boxed{2as = v^2 - u^2}$$