Oscillations

Most important question – answers

Question (1): For an oscillating body, derive an expression for its

a. Displacement

b. Velocity

c. Acceleration

d. Force acting on it

e. Time period

f. Potential energy

g. Kinetic energy

h. Total energy

Displacement

A body executing SHM can be compared with a body doing circular motion having radius A as shown. Let this body covers an angle θ in time after starting from X (A,0) at t = 0.

In ΔOBP

$$\frac{y}{A} = \sin\theta$$

$$\therefore \boxed{y = A\sin\omega t}$$

This is the displacement equation of a body amplitude of whose motion is A and angular frequency is ω

Velocity in SHM

$$v = \frac{dy}{dt} = A\omega\cos\omega t$$

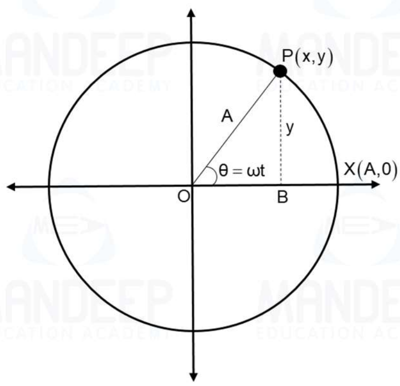
$$\Rightarrow v = A\omega\sqrt{1-\sin^2\omega t}$$

$$\Rightarrow v = A\omega\sqrt{1-\frac{y^2}{A^2}}$$

$$\Rightarrow v = A\omega\frac{\sqrt{A^2-y^2}}{A}$$

$$\Rightarrow \boxed{v = \omega\sqrt{A^2-y^2}}$$

$$v_{max} = A\omega$$



Acceleration in SHM

$$a=\frac{dv}{dt}=\frac{d}{dt}(A\omega\cos\omega t)\Rightarrow a=-A\omega^2\sin\omega t\Rightarrow \boxed{a=-\omega^2 y}\qquad ....(i)$$

Restoring force

$$F=ma$$
$$\Rightarrow \boxed{F=-m\omega^2 y}$$

Time period in SHM

$$\because F=-ky \text{ and also } F=-m\omega^2 y$$
$$\therefore k=m\omega^2$$

$$\text{So, } \omega=\sqrt{\frac{k}{m}}$$
$$\text{or } \frac{2\pi}{T}=\sqrt{\frac{k}{m}}$$
$$\Rightarrow \boxed{T=2\pi\sqrt{\frac{m}{k}}}$$

Also, from i

$$\omega=\sqrt{\frac{a}{y}}$$
$$\Rightarrow \frac{2\pi}{T}=\sqrt{\frac{a}{y}}$$
$$\Rightarrow T=2\pi\sqrt{\frac{y}{a}}=2\pi\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

Kinetic energy in SHM

$$KE=\frac{1}{2}mv^2$$
$$\Rightarrow KE=\frac{1}{2}m\left(\omega\sqrt{A^2-y^2}\right)^2$$
$$\Rightarrow \boxed{KE=\frac{1}{2}m\omega^2A^2-\frac{1}{2}m\omega^2y^2}$$

Potential energy in SHM

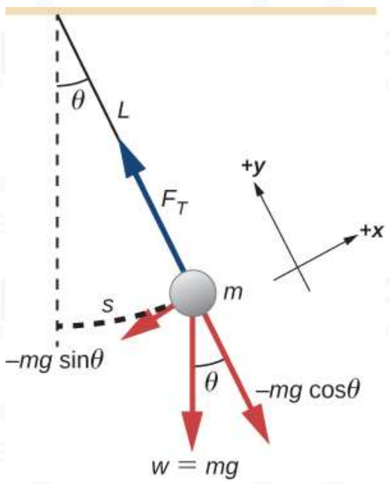
$$PE=\frac{1}{2}ky^2$$
$$\Rightarrow \boxed{PE=\frac{1}{2}m\omega^2y^2}$$

Total energy in SHM

$$TE = PE + KE \Rightarrow TE = \frac{1}{2}m\omega^2y^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2y^2 \Rightarrow \boxed{TE = \frac{1}{2}m\omega^2A^2}$$

Question (2): Derive an expression for time period of a simple pendulum.

Consider a pendulum of length L connected to a bob of mass m as shown. Now from figure it is clear that $mg\sin\theta$ provides the necessary restoring force. Therefore



$$F = -mg\sin\theta$$

for small angles $\sin\theta \approx \theta$

$$\therefore F = -mg\theta$$

$$\theta = \frac{x}{\ell}$$

$$\therefore F = -mg\frac{x}{\ell}$$

$$\text{or } F = -\left(\frac{mg}{\ell}\right)x$$

$$\therefore k = \frac{mg}{\ell}$$

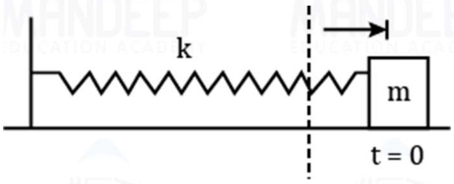
$$\text{As } T = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore T = 2\pi\sqrt{\frac{m\ell}{mg}}$$

$$\text{or } \boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

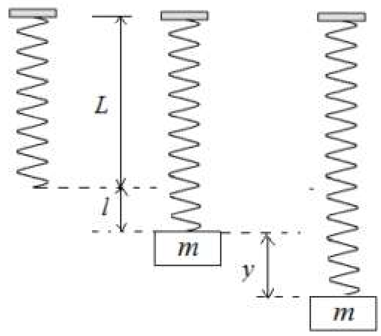
Question (3): Derive an expression for time period of a block connected to spring

Horizontal spring



In this spring mass system shown in figure above, time period of oscillation is given by

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$



Vertical spring

When a block is connected to a vertical spring, it extends by an amount ℓ so that the restoring force balances the weight of the block. Therefore,

$$mg = k\ell$$

$$\text{So, } k = \frac{mg}{\ell}$$

$$mg = k\ell$$

$$\text{So, } k = \frac{mg}{\ell}$$

Now, when this spring is pulled by a distance y, it starts doing SHM with time period T which is given by

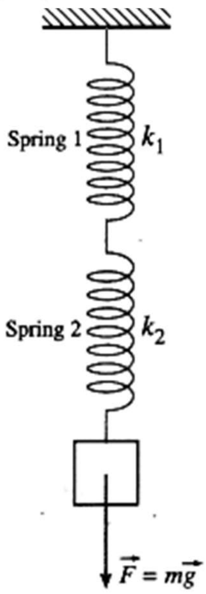
$$\therefore T = 2\pi\sqrt{\frac{m\ell}{mg}}$$

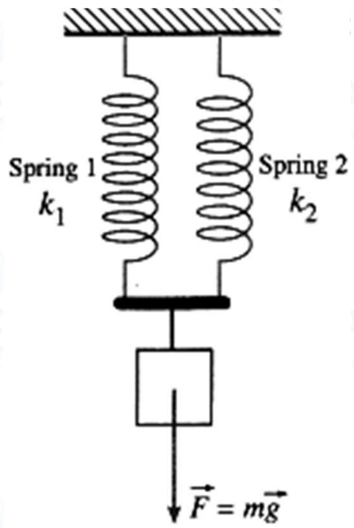
$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

Question (4): Derive equivalent spring constant for a combination of springs connected in (a) Series (b) Parallel

Series combination

Consider two springs of spring constants k_1 and k_2 connected in series as shown. Now, when this system oscillates, extensions in springs be y_1 and y_2, then





$$F = -k_1y_1 = -k_2y_2$$

Total extension is

$$y = y_1 + y_2$$

$$\Rightarrow y = -\frac{F}{k_1} - \frac{F}{k_1}$$

$$\Rightarrow y = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow F = -\left(\frac{k_1k_2}{k_1 + k_2}\right)y$$

Comparing it with $F = -kx$, we get

$$k = \frac{k_1k_2}{k_1 + k_2}$$

which gives

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}}$$

Parallel combination

Consider two springs of spring constants k_1 and k_2 connected in series as shown. Now, when this system oscillates, extensions in springs be y and restoring forces be F_1 and F_2, then

$$F_1 = -k_1 y,\ F_2 = -k_2 y$$
$$F = F_1 + F_2 = -k_1 y + -k_1 y$$
$$\Rightarrow \boxed{F = -(k_1 + k_2) y}$$
$$\therefore \boxed{k = k_1 + k_2}$$
$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}}$$