Thermodynamics

Most important questions and answers

Question (1): State zeroth’s law of thermodynamics.

The Zeroth Law of Thermodynamics states that if two systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other. In simpler terms, if two systems have the same temperature as a third system, they have the same temperature with each other. This law establishes the concept of temperature and provides the foundation for the construction of temperature scales.

Question (2): Name four types of thermodynamical processes.

Four types of thermodynamic processes are:

1. **Isothermal Process**: An isothermal process is one in which the temperature of the system remains constant throughout the process. This typically occurs when heat is exchanged between the system and its surroundings in such a way that the internal energy of the system remains constant.
2. **Adiabatic Process**: An adiabatic process is one in which there is no heat exchange between the system and its surroundings. This means that the change in internal energy of the system is solely due to work done on or by the system. Adiabatic processes are often rapid and occur without any heat transfer.
3. **Isobaric Process**: An isobaric process is one in which the pressure of the system remains constant while other parameters such as volume and temperature may change. This type of process often occurs in systems where there is constant pressure applied or when heat exchange occurs at constant pressure.
4. **Isochoric Process**: Also known as an isovolumetric or isometric process, an isochoric process is one in which the volume of the system remains constant. In such processes, the system does not perform any work on its surroundings, and all the heat added or removed changes the system's internal energy and temperature.

Question (3: What is an indicator diagram?

An indicator diagram, also known as a pressure-volume (PV) diagram, is a graphical representation that illustrates the changes in pressure and volume of a thermodynamic system during different processes. It helps visualize how the system evolves and behaves, including expansion, compression, and other transformations.

Question (4): Derive an expression for work done by an ideal gas during isothermal reversible expansion.

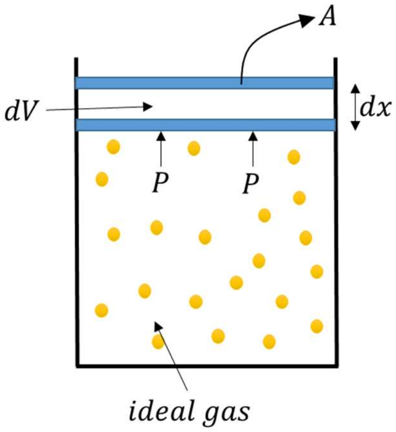
Work done in an isothermal expansion. Consider n moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in the figure below. Let P be the pressure of the gas.

Work done by the gas when the piston moves up through a small distance dx is given by

$$dW = PAdx = PdV$$

where A is the cross-sectional area of the piston and $dV = Adx$, is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state (P_1, V_1) to the final state (P_2, V_2). The total amount of work done will be

$$W_{iso} = \int_{V_1}^{V_2} PdV$$



For n moles of a gas, PV = nRT or $P = \frac{nRT}{V}$

$$\therefore W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV = nRT \, [\ln V]_{V_1}^{V_2}$$

$$= nRT \, [\ln V_2 - \ln V_1] = nRT \ln \frac{V_2}{V_1}$$

or $\boxed{W_{iso} = 2.303 \, nRT \log \frac{V_2}{V_1} = 2.303 \, nRT \log \frac{P_1}{P_2}}$

Question (5): Derive an expression for work done by an ideal gas during adiabatic expansion.

Work done in an adiabatic expansion. Consider n moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let P be the pressure of the gas. When the piston moves up through a small distance dx, the work done by the gas will be

$$dW = PAdx = PdV$$

where A is the cross-sectional area of the piston and $dV = Adx$ is the increase in the volume of the gas.

Suppose the gas expands adiabatically and changes from the initial state (P_1, V_1, T_1) to the final state (P_2, V_2, T_2). The total work done by the gas will be

$$W_{adia} = \int_{V_1}^{V_2} PdV$$

For an adiabatic change $PV^{\gamma} = K$ or $P = KV^{-\gamma}$, $\therefore$

$$W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma}dV$$

$$= K\int_{V_1}^{V_2} V^{-\gamma}dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma-1}[KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

But $K = P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}]$$

$$W_{adia} = \frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$$

Also, $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$

$$\boxed{W_{adia} = \frac{nR}{\gamma-1}[T_1 - T_2]}$$

Question (6): What is the equation of state for Isothermal process and adiabatic process?

Isothermal process:

Equation of state for an isothermal process is PV = constant. i.e. the product of pressure and volume for is always constant at constant temperature.

Adiabatic process:

Equation of state for an isothermal process is

$PV^{\gamma} = \text{constant}$

Where $\gamma = \frac{C_p}{C_v}$; C_p = molar heat capacity at constant pressure, C_v = molar heat capacity at constant volume.

Question (7): Explain why and adiabatic curve is steeper than an isothermal curve?

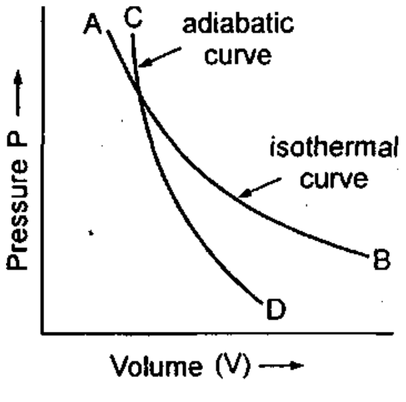


Fig. 20·1

As we know, slope is $\frac{dP}{dV}$

For isothermal process,

$PV = k$

differentiating both sides, we get

$PdV + VdP = 0$

$\Rightarrow \frac{dP}{dV} = -\frac{P}{V}$

For adiabatic process, we have

$$PV^{\gamma} = k$$

differentiating both sides, we get

$$P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0$$

$$\Rightarrow \frac{dP}{dV} = -\gamma\frac{P}{V}$$

Clearly slope of adiabatic curve is gamma times more that slope of isothermal curve and since gamma is always greater than 1, so slope of adiabatic curve is more than that of isothermal curve.

Hence an adiabatic curve is always steeper than an isothermal curve.

Question (8): What is first law of thermodynamics. State in mathematically also.

The First Law of Thermodynamics is a statement of the conservation of energy applied to any system in which energy transfer from or to the surroundings is taken into account. It states that the heat given to a system is either used in doing external work or it increases the internal energy of the system, or both.

In formula terms, this is expressed as:

$$\boxed{\Delta Q = \Delta U + \Delta W}$$

where,

- ΔQ = Heat supplied to the system by the surroundings.
- ΔW = Work done by the system on the surroundings.
- ΔU = Change in the internal energy of the system.

Question (9): State and prove Mayer's formula or derive a relation between two principle specific heats of a gas.

Consider n moles of an ideal gas. Heat the gas to raise their temperature by dT. According to the first law of thermodynamics, the heat supplied dQ is used to partly to increase the internal energy and partly in doing the work of expansion. That is,

$$dQ = dU + PdV$$

If the heat dQ is absorbed at constant volume, then dV = 0 and we have

$$dQ = nC_V dt \text{ and } dQ = dU$$

$$\therefore dU = nC_V dt \quad \text{.................(i)}$$

If now the heat dQ is absorbed at constant pressure, then

$$dQ = dU + PdV$$

$$\Rightarrow nC_P dt = dU + PdV$$

Change in internal energy is same in both case because temperature change is same.

Using (i) we get

$$nC_Pdt = nC_Vdt + P\Delta V$$
$$\Rightarrow n\left(C_P - C_v\right)dt = PdV$$

$$\because PV = nRT$$
$$\therefore PdV = nRdT$$

Putting this in above relation, we get

$$n\left(C_P - C_v\right)dt = nRdt$$
$$\text{or}\quad C_P - C_V = R$$

This is the required relation between C_P and C_V. It is also known as Mayer's Formula.

Question (10): What are limitations of first law of thermodynamics?

Two main limitations of the first law of thermodynamics are:

1. It does not provide information about the directionality or reversibility of processes.
2. It does not explicitly account for changes in entropy, which is crucial for understanding the directionality and irreversibility of processes.

Question (11): Write various statements of second law of thermodynamics.

1. **Clausius Statement**: Heat cannot spontaneously flow from a colder body to a hotter body without the input of external work.
2. **Kelvin-Planck Statement**: It is impossible to construct a device that operates in a cycle and produces no effect other than the extraction of heat from a single thermal reservoir and the performance of an equivalent amount of work.

Question (12): Explain the working of Carnot engine.

The Carnot engine, conceptualized by Sadi Carnot in 1824, represents an idealized heat engine operating between two temperature reservoirs: a high-temperature source (T_1) and a low-temperature sink (T_2). This theoretical engine is significant for its role in thermodynamics, serving as a benchmark for evaluating the efficiency of real heat engines. Let's delve into its components and the Carnot cycle.

Components of the Carnot Engine:

1. **Cylinder:**
    - Contains an ideal gas as the working substance.
    - Features conducting base and insulating walls.
    - Equipped with an insulating, frictionless piston.
2. **Source (Hot Reservoir):**

  - Maintained at a constant high temperature (T_1).
  - Designed with conducting walls.
  - Has infinite thermal capacity, allowing heat extraction without temperature change.

3. **Sink (Cold Reservoir):**
   - Kept at a constant lower temperature (T_2).
   - Also has infinite thermal capacity, enabling heat rejection without altering its temperature.
4. **Working Substance:**
   - Typically, an ideal gas used within the cylinder.
5. **Insulating Stand:**
   - Used to isolate the cylinder's base from the surroundings, preventing heat transfer.

The Carnot Cycle: The working substance in the Carnot engine undergoes a series of reversible processes known as the Carnot cycle. This cycle consists of two isothermal processes (heat transfer at constant temperature) and two adiabatic processes (no heat transfer):

1. **Isothermal Expansion (at T_1):** The gas expands isothermally while absorbing heat from the hot reservoir.
2. **Adiabatic Expansion:** The gas continues to expand without heat exchange, reducing its temperature to T_2, the temperature of the cold reservoir.
3. **Isothermal Compression (at T_2):** The gas is compressed isothermally, releasing heat to the cold reservoir.
4. **Adiabatic Compression:** The gas is further compressed without heat transfer, which increases its temperature back to T_1.

During this cycle, the engine converts a portion of the heat absorbed from the hot reservoir into work while expelling the remaining heat to the cold reservoir. The efficiency of a Carnot engine is fundamentally dependent on the temperatures of the hot and cold reservoirs and is given by:

Efficiency $\boxed{\eta = 1 - \frac{T_2}{T_1}}$

where T_1 and T_2 are the absolute temperatures of the hot and cold reservoirs, respectively.

The Carnot engine, while theoretical and unachievable in practice, plays a critical role in thermodynamics. It sets an upper limit on the efficiency that any real heat engine can achieve, serving as a fundamental guide in understanding the principles of energy transfer and the limitations imposed by the second law of thermodynamics.

Question (13): Show that efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$, where T_2 and T_1 are temperatures of sink and source respectively.

In a Carnot engine, first step in isothermal expansion. Let the volume of n moles of gas increases from V_1 to V_2 at temperature T_1, then work done by the gas is

$Q_1 = W_1 = 2.303nRT_1 \log\frac{V_2}{V_1}$, where Q_1 is the heat gained by system

Second step is adiabatic expansion, now let the volume of gas increases from V_2 to V_3 and temperature changes from T_1 to T_2, then work done is

$$W_2 = \frac{1}{1-\gamma}(T_2 - T_1)$$

Third step is isothermal compression, let the volume of gas changes from V_3 to V_4, then work done is

$Q_2 = W_3 = -2.303nRT_2 \log\frac{V_4}{V_3}$, where Q_2 is the heat loss by the system.

Step 4 is adiabatic compression in which the volume V4 changes back to intial volume V1, then work done is

$$W_4 = -\frac{1}{1-\gamma}(T_1 - T_2)$$

Net work done is

$$W_{net} = W_{exp} - W_{com} = W_1 + W_2 - (W_3 + W_4)$$

since $W_2 = W_4$

$$W_{net} = W_1 - W_3 = Q_1 - Q_2$$

Note: You can directly start this derivation from above step also, you can consult your school teacher

Also

For step 1, we can write

$P_1V_1 = P_2V_2$ ....(i)

For step 2

$P_2V_2^{\gamma} = P_3V_3^{\gamma}$ ......(ii)

For step 3

$$P_3V_3 = P_4V_4 \qquad \text{...........(iii)}$$

For step 4

$$P_4V_4^{\gamma} = P_1V_1^{\gamma} \qquad \text{.......(iv)}$$

Therefore, we have

$$\cancel{P_1P_2P_3P_4}\,V_1V_2^{\gamma}V_3V_4^{\gamma} = \cancel{P_1P_2P_3P_4}\,V_1^{\gamma}V_2V_3^{\gamma}V_4$$

$$\left(V_1V_3\right)^{1-\gamma} = \left(V_2V_4\right)^{1-\gamma}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{V_4}{V_3} \qquad \text{.......(v)}$$

Now,

$$\frac{Q_2}{Q_1} = \frac{-\cancel{2.303nR}T_2\log\left(\frac{V_4}{V_3}\right)}{\cancel{2.303nR}T_1\log\left(\frac{V_2}{V_1}\right)}$$

$$\frac{Q_2}{Q_1} = -\frac{T_2}{T_1}\frac{\log\left(\frac{V_4}{V_3}\right)}{\log\left(\frac{V_2}{V_1}\right)} = \frac{T_2}{T_1}\frac{\log\left(\frac{V_4}{V_3}\right)^{-1}}{\log\left(\frac{V_2}{V_1}\right)} \qquad [\because a\log b = \log b^a]$$

$$\therefore \frac{Q_2}{Q_1} = \frac{T_2}{T_1}\frac{\log\left(\frac{V_3}{V_4}\right)}{\log\left(\frac{V_2}{V_1}\right)}$$

Since $\frac{V_3}{V_4} = \frac{V_1}{V_2}$ [using (v)]

$$\therefore \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Since

$$\eta = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \boxed{\eta = 1 - \frac{T_2}{T_1}}$$

Question (14): Show that efficiency of a heat engine is $1-\frac{Q_2}{Q_1}$, where Q_1 is heat supplied by source and Q_2 is heat given out into the sink.

Let a working substance in a heat engine absorbs heat Q_1 from a source and rejects heat Q_2 into the sink. So, heat used for performing work is $Q_1 - Q_2$. This must be equal to net work done by the working substance. Hence $W_{net} = Q_1 - Q_2$.

$$\because \eta = \frac{\text{Output work}}{\text{Energy absorbed}}$$

$$\therefore \eta = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \boxed{\eta = 1 - \frac{Q_2}{Q_1}}$$