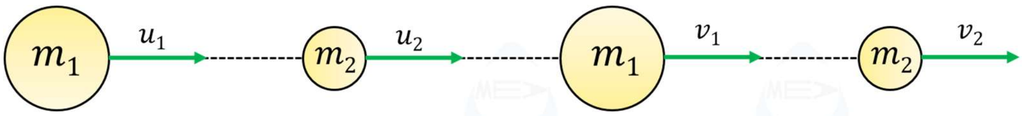
Work Energy Power

Most important questions with answers

Note: Questions are numbered according to their importance

Question (1): Discuss elastic collision in one dimension. Derive an expression for velocities of two bodies after such a collision.

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 moving in the same straight line colliding with each other. Let their velocities be v_1 and v_2 after the collision.



Since momentum remains conserved in an elastic collision, therefore

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\Rightarrow m_1u_1 - m_1v_1 = +m_2v_2 - m_2u_2$$
$$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ....(i)$$

As kinetic energy is also conserved in elastic collision therefore

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$\Rightarrow \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$
$$\Rightarrow \frac{1}{2}m_1(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$$
$$\Rightarrow \cancel{\frac{1}{2}}m_1(u_1 - v_1)(u_1 + v_1) = \cancel{\frac{1}{2}}m_2(v_2 - u_2)(v_2 + u_2).......(ii)$$

From (i) and (ii), we get

$$\frac{\cancel{m_1}\cancel{(u_1 - v_1)}(u_1 + v_1)}{\cancel{m_1}\cancel{(u_1 - v_1)}} = \frac{\cancel{m_2}\cancel{(v_2 - u_2)}(v_2 + u_2)}{\cancel{m_2}\cancel{(v_2 - u_2)}}$$
$$\Rightarrow u_1 + v_1 = v_2 + u_2$$
$$\Rightarrow u_1 - u_2 = v_2 - v_1 \quad ....(iii)$$

Thus, velocity of approach = velocity of separation. Since

$$e(\text{coefficient of restitution}) = \frac{\text{velocity of seperation}}{\text{velocity of approach}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Therefore, for perfectly elastic collision, e = 1

Now, from (iii), we get

$v_2 = u_1 - u_2 + v_1$, putting this in momentum conservation equation, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$
$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$
$$\Rightarrow (m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$
$$\Rightarrow \boxed{v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)}}$$

Similarly, we can prove that

$$\boxed{v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1u_1}{(m_1 + m_2)}}$$

Question (2): Derive an expression for the elastic potential energy of a stretched spring.

Consider a spring of spring constant k. Let one end of this spring is fixed and a force F is applied on the other end to stretch its length by small amount dx. Then, work done is

$$dW = \vec{F}.\vec{dx}$$
$$\Rightarrow dW = -(kx)dx\cos 180^\circ$$
$$\Rightarrow dW = -kx(-1)dx$$
$$\Rightarrow dW = kxdx$$

Total work done in stretching the spring from 0 to x_o

$$W = \int_0^{x_o} kxdx$$
$$\Rightarrow W = k\int_0^{x_o} xdx$$
$$\Rightarrow W = k\left[\frac{x^2}{2}\right]_0^{x_o}$$
$$\Rightarrow W = \frac{k}{2}\left[x_o^2 - (0)^2\right]$$
$$\Rightarrow W = \frac{1}{2}kx_o^2$$

This work is stored in the spring in the form of elastic potential energy, so

$$\boxed{U = \frac{1}{2}kx_o^2}$$

Question (3): State and prove the work energy theorem.

Consider a body of mass m moving with a velocity v_i. Now let a force F is applied to it and its velocity becomes v_f after some time. Let the velocity change be dv for small time dt and body travels a distance ds during this time, then work done dW is

$$dW = Fds$$
$$\Rightarrow dW = (ma)ds$$
$$\Rightarrow dW = m\frac{dv}{dt}ds$$
$$\Rightarrow dW = mdv\left(\frac{ds}{dt}\right)$$
$$\Rightarrow dW = mvdv$$

So, total work done when velocity changes from v_i to v_f

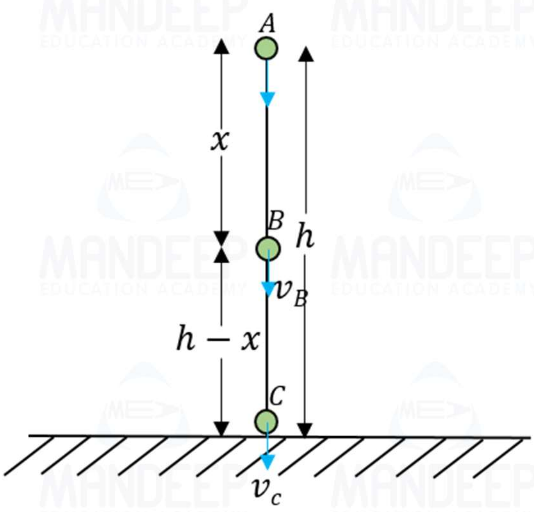
$$\Rightarrow \int dW = \int_{v_i}^{v_f} mvdv$$
$$\Rightarrow W = m\left[\frac{v^2}{2}\right]_{v_i}^{v_f}$$
$$\Rightarrow W = m\left[\frac{v_f^2}{2} - \frac{v_i^2}{2}\right]$$
$$\Rightarrow W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Question (4): Prove that a body falling freely the total mechanical energy always remains conserved.

Consider a body of mass m falling form a height h. Consider three points A, B and C in its path as shown. Now

Total energy of body at A is only potential as its velocity is zero. Therefore

$$T.E_A = mgh$$



At B, body has both potential and kinetic energy. Since the body has covered a distance x, therefore its velocity at B is

$$v_B^2 - (0)^2 = 2gx$$
$$\Rightarrow v_B^2 = 2gx$$

So total energy of body at B is

$$TE_B = mg(h-x) + \frac{1}{2}mv_B^2$$
$$\Rightarrow TE_B = mg(h-x) + \frac{1}{2}m(2gx)$$

$$\Rightarrow TE_B = mgh - mgx + mgx$$
$$\Rightarrow TE_B = mgh$$

At point C, body has only kinetic energy. Velocity of body at C is

$$v_C^2 - (0)^2 = 2gh$$
$$\Rightarrow v_C^2 = 2gh$$

So, total energy of body at C is

$$TE_C = \frac{1}{2}mv_C^2$$
$$\Rightarrow TE_C = \frac{1}{2}m(2gh)$$
$$\Rightarrow TE_C = mgh$$

Clearly, $TE_A = TE_B = TE_C$

Question (5): Derive a formula for kinetic energy of a body of mass m moving with velocity v.

Consider a body of mass m kept at rest. Now let a force F is applied to it and its velocity becomes v after some time. Let the velocity change be dv for small time dt and body travels a distance ds during this time, then work done dW is

$$dW = Fds$$
$$\Rightarrow dW = (ma)ds$$
$$\Rightarrow dW = m\frac{dv}{dt}ds$$
$$\Rightarrow dW = mdv\left(\frac{ds}{dt}\right)$$
$$\Rightarrow dW = mvdv$$

So, total work done in change velocity from 0 to v is

$$W = \int_0^v mvdv$$
$$\Rightarrow W = m\left[\frac{v^2}{2}\right]_0^v$$
$$\Rightarrow W = \frac{1}{2}m\left(v^2 - (0)^2\right)$$
$$\Rightarrow W = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 \Rightarrow \boxed{W = \frac{1}{2}mv^2}$$

This work is stored in the body in the form of Kinetic energy. This $KE = \frac{1}{2}mv^2$

Question (6): Derive an expression for common velocity of two bodies after perfectly inelastic collision.

Consider two bodies of masses m_1 and m_2 moving with velocities v_1 and v_2 collide inelastically and stick together with each other. Let their common velocity after collision be v. As momentum remains conserved in inelastic collision, therefore,

$$m_1v_1 + m_2v_2 = (m_1v + m_2v)$$

$$v(m_1 + m_2) = m_1v_1 + m_2v_2$$

$$\boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

Question (7): Derive an expression for loss of energy when two bodied of masses m1 and m2 moving with velocities v1 and v2 undergo inelastic collision.

Total KE of the bodies before collision

$$KE_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Total KE after collision

$$KE_f = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 \text{ (as velocity becomes equal after inelastic collision)}$$

Therefore, loss of kinetic energy is

$$KE_i - KE_f$$

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1v_1 + m_2v_2}{m_1 + m_2}\right)^2$$

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}\cancel{(m_1 + m_2)}\frac{(m_1v_1 + m_2v_2)^2}{(m_1 + m_2)^{\cancel{2}}}$$

$$= \frac{1}{2}\left[\frac{m_1v_1^2(m_1 + m_2) + m_2v_2^2(m_1 + m_2) - (m_1^2v_1^2 + m_2^2v_2^2 + 2m_1m_2v_1v_2)}{(m_1 + m_2)}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{m_1^2v_1^2} + m_1m_2v_1^2 + m_1m_2v_2^2 + \cancel{m_2^2v_2^2} - \cancel{m_1^2v_1^2} - \cancel{m_2^2v_2^2} - 2m_1m_2v_1v_2}{(m_1 + m_2)}\right]$$

$$= \frac{1}{2}m_1m_2\left[\frac{(v_1^2 + v_2^2 - 2v_1v_2)}{(m_1 + m_2)}\right] \Rightarrow \boxed{\Delta KE = \frac{1}{2}m_1m_2\frac{(v_1 - v_2)^2}{(m_1 + m_2)}}$$